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Many and sincere thanks to all of those friends, students, colleagues, and faculty members (all fall into the first category) who put up my ongoing requests for feedback on the material in this manual. Many thanks to those who responded, whether verbally or in writing.

This manual was not written from scratch; the previous version(s) of the manual provided a solid foundation for me to work from. Thanks to all previous authors of this manual, and those who supported them.

The material that I added was based on my experience over several years teaching the Control Systems Lab. I tried to recall the places where students had difficulty understanding the procedures, and where the lecture was slightly behind the information need for the lab exercise. Thanks to all of my students from previous lab classes for helping me to formulate this material by not being too shy to ask “What?” and “Why?”, and to those who dug into the course material to figure it out when I didn’t have the answers.

Finally, thanks to my family for putting up with the late night and early morning editing sessions, the stacks of Control Systems reference books, and assorted scatters of papers.

I was consumed to some extent with this project, but I am very satisfied with the results. I hope that the “consumers” of this product are satisfied as well.

Benjamin D. Sweet
December 22, 2003
References

Introduction

This laboratory manual describes the exercises to be performed in the laboratory of the Control Systems Laboratory course, offered by the ECE Department at Lawrence Technological University.

The laboratory exercises focus on a linearized DC servomotor that has a power rating below one horsepower. The motor is linearized by using feedback concepts implemented by amplifiers and the tachometer generator. This assembly is shown under “Servo Motor System” in the “Lab Apparatus” section.

Associated equipment includes a potentiometer for angular position sensing and a DC generator (tachometer) for angular velocity sensing. Other equipment includes a power supply, operational amplifier, pre-amplifier, servo amplifier, and a dual attenuator (potentiometer). This equipment is pictured and further described in the “Lab Apparatus” section. The equipment is manufactured by Feedback, Inc. of the U.K.

Input signals to the subject systems are produced from either a function generator or a variable DC voltage. Outputs from the systems are observed either on an oscilloscope or from DC voltages measured on a volt-meter.

The primary data observation device is a Digital Storage Oscilloscope (DSO). The data may be captured by sending the image on the oscilloscope screen to a printer, or storing the image on a disc. This permits recording data in the usual way by oscilloscope and then obtaining a hard copy by a screen dump option. Instructions for the basic operation of the oscilloscope for these labs are discussed in “Appendix – TDS3012 Oscilloscope Specifications.”

The exercises normally performed for the course are listed in the next section. The schedule includes the expected number of lab sessions necessary to complete the laboratory classroom portion of the lab.

Students should utilize simulation programs such as MatLab/Simulink to assist in the design and analysis of the exercises. The simulation programs will also provide an additional tool to assist in understanding the concepts studied. MatLab/Simulink is available in a student PC edition; MatLab is also available on the University computer system. MathCad may be used in the algebraic computation of system parameters, theoretical values, and expected results. Spreadsheet software, such as Microsoft Excel, may be used to tabulate and process data collected during the lab exercises.

Laboratory reports are required of all exercises performed in the course. Suggestions for content and format are discussed in “Lab Report Format.”
# Lab Exercise Schedule

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<td></td>
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</tr>
</tbody>
</table>

*This schedule assumes accommodation for extra lab time (if needed) and for the instructor’s option to have in-lab examinations.*
Lab Report Format

The lab instructor may have specific requirements regarding lab report content and format; if so, these requirements should be clearly specified in the class syllabus or accompanying introductory class material.

Where not specified by the instructor, use your own judgment as to the presentation of the material. Here are a few suggestions:

General
Each student is responsible for submitting an individual laboratory report based on his or her own work. A small team of 2-3 people can perform the exercises.

It is important that the student attach engineering units to key results throughout the report. (e.g., use 3.0 Volt/rpm instead of just 3.0).

The laboratory report should contain enough information so that it can stand alone without reference to the laboratory manual.

On Report “Completeness”
Write your lab report as if you were presenting it to somebody who was not involved in the lab exercise; it is reasonable to assume that the reader is familiar with the lab equipment. Consider whether they would be able to follow and understand your report without referring to the lab manual.

Include supporting documentation (such as diagrams, tables, etc.) that is needed to understand your report. Label all diagrams, tables, etc., and refer to them by their labels in the body of the report.

Explain what you intend to do (background, lab set-up, schematics), what you expect to observe (pre-lab, theoretical calculations), what you actually observed (actual measured values, percent error analysis where appropriate), and your conclusions (did you observe what you expected, and why or why not?)

Try to organize the report in a manner so as to minimize “flipping” back and forth between the body of the report and the supporting documentation. If graphs, oscilloscope traces, computations, etc. are included as appendices, summarize the results in the body of the report, and then refer to the label of the supporting documentation.
On Report Organization

**Title Sheet:**
This page should contain the exercise number and title, the name of all students in the group, the date(s) that the exercise was conducted, and the date that the report was submitted. The author’s name should be clearly identified. This section should also include any pertinent equipment identification, such as motor serial number; the same motor assembly should be used throughout the semester to minimize variability.

**Purpose:**
The next page should contain a short original paragraph of the student’s own interpretation of the purpose of the laboratory exercise. Enough detail should be included so that the laboratory report can stand on its own without reference to the laboratory manual.

**Pre-Lab Work:**
Specific preparations are necessary in order to make the laboratory class work progress smoothly and efficiently. This section should briefly explain what theoretical values are being designed or calculated. Design work often requires a large amount of algebraic manipulation in order to determine component values and other system parameters needed to satisfy the design criteria, or to predict the subject system’s behavior. These pre-lab calculations should be included in the final lab report, even if they are in hand-written form. They may be included within this section of the report, or included as an appendix section and referred to in this section of the report along with a summary of the computed values. CAE (Computer Aided Engineering) software may be used here; Matlab/Simulink and MathCad are available on the University computer system. This section should be completed in enough detail so that the student can carry out the exercise efficiently in the laboratory; the instructor may ask to review the pre-lab material during the laboratory period.

**Lab Setup and Procedure:**
This section contains a brief description of the lab equipment setup and procedure for carrying out the lab exercise. It is not necessary to restate the entire procedure from the lab manual, but a brief description that could be followed by someone familiar with the lab equipment should be given. Reference included block diagrams, wiring diagrams, schematics, etc. Also mention other lab equipment settings and connections.

For example:

Perform the Balance Pre-Amp Output Procedure and Zero Op-Amp Output Procedure.

**Note:** It is not necessary to repeat the explanation of these procedures in EACH lab report, but a brief summary of the procedure should be included in the report for the first exercise in which the procedure is used. Then, in subsequent lab reports, it should be sufficient to just say “Perform the xyz procedure.”

Connect the Op-Amp, Dual Attenuator, Pre-Amp, Servo-Amp, Motor, Tachometer, and Position Indicator as shown in Figure 1; the corresponding block diagram is shown in Figure 2. The Op-Amp acts as the summing junction; the Dual Attenuator
is used for the variable gain, $K_a$. Connect the signal generator to plug 1 of the Op-Amp unit; this serves as the input voltage, $V_{in}$. Connect Channel 1 of the Oscilloscope to monitor $V_{in}$, and Channel 2 to monitor $V_{position}$.

Etc...

**Results:**
This section contains a summary of the key results and observations. The measured (observed) results, comparison to the theoretical values or simulated results, and discussion of the error analysis should be included here. If data is collected in an appendix section, it should be referred to here. If error analysis computation is contained in an appendix section, it should be referred to here along with a summary of the error values.

**Conclusions and Comments:**
This section contains closing remarks summarizing the key results and observations from the lab exercise. The student should include a discussion of what was learned from the exercise. Explanations of the results, good or bad, and the reasons for error are some topics.

Answers to specific questions in the lab manual should be included; the questions themselves should be included or implied in the statement of the answers. The answers to the questions should be complete, coherent, and generally supported by the overall lab report.

Any deviations from the procedures in the lab manual should be discussed here along with the reasons for deviating from the procedure (perhaps lab equipment was not functioning properly or was not available, etc.)

The student should discuss the results of the lab exercise and explain significant deviations between the measured (observed) values versus theoretical values, based on the error analysis. Do not use qualitative words like “close” or “good” or “successful” without some supporting quantitative information. (Refer to Error Analysis section.)

For example:

All but two of the measured values were within 5% of the expected values, and all were within 10% of expected values. It is therefore concluded that the theoretical analysis accurately predicted the observed system behavior.

(Use your own original statement.)

**Note:** Errors between expected and observed values do not constitute “failure” to successfully perform the lab exercise. There are many reasons for errors between expected and observed values that are not attributable to “operator error.” Yes, double-check your math in the derivation of expected values, but also consider the many other possible sources of error. For example, the model for the D.C. motor that is used in the computation of expected values ignores armature inductance, changes in armature resistance with temperature of the motor, and the non-linear effects of motor speed saturation and gear backlash.
The following may be included as appendices to the report, or included at the appropriate place(s) throughout the report. If they are included as appendices, refer to the relevant labeled appendix section or result from within the body of the report, and also include the final result (along with the reference) in the body of the report.

**Calculations:**
This section provides the computational framework. This section should include the origin and derivation of the key equations used in the analysis and design. In this section the laboratory data should be reduced to the key parametric results required. The error analysis should be included. Error analysis involves comparing the measured (observed) values or results with the expected, or theoretical, values.

**Diagrams and Schematics:**
This section will include a block diagram representation of the system. The name and transfer function of each block in the diagram should be included. Also, the signals that connect the blocks should be identified (labeled). If discrete components are used, their assembly should be shown in a separate sub-schematic in the block diagram. Overall lab setup wiring diagrams may be included here as well.

**Data:**
This section should contain raw data obtained in the exercise, such as hard copies of time response (“traces”) from the oscilloscope, data tables, and recorded measurements. Sample calculations used in the data reduction should be included here, or refer to the relevant information in the calculation section. All graphs should be clearly labeled. This means the axes should have variable names and engineering units and that key parametric features, such as rise time or overshoot, should be highlighted on each graph. This may involve annotating the original raw data hard copy.

It is reasonable to use spreadsheet software, such as Microsoft Excel, to tabulate data and compute subsequent values. If formulas are used within cells of a spreadsheet, include in the report a sample of the calculation showing the formula and referring to the cells in the spreadsheet that use the formula.
**Terminology**

$M_p$  Percent Overshoot (also referred to as %OS) – The amount that the peak value of an under-damped step response overshoots the final value, expressed as a percentage of the final value.

$t_p$  Peak-Time – The amount of time from the beginning of an under-damped step response to the response peak value.

$t_r$  Rise-Time – In an under-damped step response, the amount of time between the response first reaching 10% of the response final value and the response first reaching 90% of the final value.

$t_s$  Settling-Time – In an under-damped step response, the amount of time from the beginning of the step response until the response oscillation amplitude remains within a specified range of the response final value. The settling range (“band”) is typically expressed as a percentage of the final value. A 2% settling band has a range of ± 1% around the final value.
Error Analysis

Accuracy, Precision & Resolution
Some values are exact:

- 1000 meters in one kilometer
- 60 seconds in one minute
- 12 eggs in one dozen

Other values (such as measurements or estimations) have some degree of uncertainty.

**Accuracy** is the measure of *how True or Correct* the scale of a measurement is.  
**Precision** is the measure of *Repeatability* of a measurement (without the measurement necessarily being correct.)  Precision could be evaluated for the same person taking the measurement several times, or for different people taking the same measurement.  
**Resolution** is the level of *Fineness* of a measurement, or the ability to distinguish between two measurement points.

For example, which would be a “better” measurement for 1 centimeter?

- Using a meter-stick that is EXACTLY 1 meter long (Perfect Accuracy!) that has NO intermediate markings (Terrible Resolution!).  How repeatable (Precise) would this measurement be?
- Using a ruler that was the prize from a box of cereal (Questionable Accuracy) that is marked off in centimeters (Perfect Resolution!).  How repeatable (Precise) would this measurement be?

Digital meters are ASSUMED to be Accurate; regular calibrations help to assure this.  
Digital meters have a different number of digits after the decimal point (Resolution) based on the *scale* or *range* of the measurement (up to 1 Volt, up to 10 Volts, etc.)

Evaluation of Measurement Error and Deviation

"The difference between theory and practice is that, in theory, there is no difference between theory and practice, and in practice there usually is." – Author Unknown

When measurements are taken in the lab, the “actual” or “measured” values observed almost never equal the “expected” or “theoretical” values predicted from the calculations.  
This may be due to many factors:

- Tolerance in manufactured parts (5% resistors, etc.)
- Unanticipated or un-accounted for resistance, inductance, and capacitance in wires and connectors
- Resistance or “loading” from the measurement equipment
- Errors in taking and/or reading the measurement (accuracy, precision, resolution)
• “Round-Off” Error in the calculation of the theoretical value
• Inaccuracies in the model used to derive the theoretical value (model simplifications, etc.)
• etc., etc., …

To quantify how closely reality (“measured”) comes to expectation (“theoretical”), a percent error analysis is performed:

\[
\%\text{Error} = \left| \frac{\text{Theoretical} - \text{Measured}}{\text{Theoretical}} \right| \times 100\% = \left| \frac{\text{Error}}{\text{Theoretical}} \right| \times 100\%
\]

It is necessary to divide the difference between the “theoretical” and “measured” values (the error) by the “theoretical” value to normalize the error. For example, if a theoretical value is 100V and the measured value is 90V, the error is 10V; if a theoretical value is 5V and the measured value is 4V, the error is 1V. Which error is “worse?” Certainly the 10V error has a larger magnitude, but it only deviates from the theoretical value by 10% while the 1V error deviates from its theoretical value by 20%. Therefore, one could say that the 1V error is more severe than the 10V error.

**Deviation Analysis**

In cases where a parameter is measured more than once or by more than one method, it is possible to analyze how closely the different measurements relate to one another. The term “error analysis” is not applicable since no single measurement can really be considered as the “True” value.

Several different methods may be used to evaluate the deviation of the different measurements from one another:

- For two measurements – M₁ & M₂ :
  - Difference:
    \[
    \%\text{Difference} = \left| \frac{M₁ - M₂}{\text{Min}(M₁, M₂)} \right| \times 100\%
    \]

- For a set of measurements – \{M₁ … Mₙ\} :
  - Deviation from the Mean for one of the measurements, Mᵢ:
    \[
    \%\text{Mean Deviation} = \left| \frac{\text{Mean}\{M₁…Mₙ\} - Mᵢ}{\text{Mean}\{M₁…Mₙ\}} \right| \times 100\%
    \]
  - Range Deviation:
    \[
    \%\text{Range Deviation} = \left| \frac{\text{Max}\{M₁…Mₙ\} - \text{Min}\{M₁…Mₙ\}}{\text{Min}\{M₁…Mₙ\}} \right| \times 100\%
    \]
Lab Apparatus

The Feedback, Inc. servo motor system modular components are:

- Power Supply
- Servo Amplifier
- Motor
- Tachometer/Generator (with Digital Volt Meter)
- Position Indicator (1 Input/ 1 Output)
- Pre-Amplifier
- Operational Amplifier
- Dual Attenuator
- Function Generator

Elements can be switched between lab stations except for the Motor Generator. The same Motor Generator should used for all lab exercises, otherwise the Km and τm parameters will be inconsistent, as motor properties are similar but not identical. Using the same motor will reduce experimental error. RECORD THE MOTOR INVENTORY TAG NUMBER, AND CHECK IT AT EVERY SESSION.

Certain “umbilical” connections are made between the Power Supply and Servo Amplifier in the rear of the boxes and from the Servo Amplifier to the Motor.
- ii - Motor, Tachometer, and Position Indicator

- iii - Dual Attenuator, Pre-Amp, and Op-Amp
- iv - Position Indicator Face Plate
**Servo Motor System**

**Motor Linearization**
The motor is a DC constant field, armature current controlled motor. Its properties are not completely linear. To improve the linearity, the Pre-Amp, Servo-Amp, Motor, and Tachometer are used and modeled as ONE UNIT. This assembly forms the core of each subject system, and must be assembled at the beginning of each lab session. The connections are shown in Figure 1.

![Figure 1 - Motor Linearization Connections](image)

The Pre-Amp has two controls on the faceplate: a three-position switch, which is set to defined $\tau$, and a Zero Set dial. Also, at the top of both the Pre-Amp and Tachometer units are three power connections for $+15V$, $-15V$, and ground. This D.C. power can be provided either from the Servo-Amp or the Power Supply. (Note that the Servo-Amp and the Power Supply share the D.C. power sources by means of the rear “umbilical” connection between them.)

At the start of every lab session, the **Balance Pre-Amp Output Procedure** (refer to Procedure section, page 31) must be performed to balance the differential output signal of the Pre-Amp unit.
**Tachometer**

The Tachometer unit contains a tachometer/generator, a 1/30 gear reduction, a digital read-out of RPM, and a digital voltmeter read-out. The Tachometer unit requires +15V, -15V, and ground connections from either the Servo-Amp or the Power Supply.

The tachometer is a D.C. generator connected to the high RPM (motor) side of the gear. The negative potential (plug 1) is used as the velocity feedback to the system; the positive potential (plug 2) should be connected to ground.

The Tachometer unit contains two digital read-outs that share a common display; a two-position switch selects which quantity is displayed.

- For a digital read-out (in RPM) of a frequency input on “tacho rpm” (plug 3), slide the two-way selector switch to the left (toward plug 3) and connect the signal to me measured to plug 3. Attaching plug 1 to plug 3 gives the motor speed in RPM.
- For a digital read-out of a D.C. Voltage (+20V MAX) on “dc volts” (plug 4), slide the two-way selector switch to the right (toward plug 4) and connect the signal to me measured to plug 4.

**Dual Attenuator**

The **Dual Attenuator** unit consists of two 10KΩ potentiometers.

The marks around the dials are only meant as a guide to the true setting. It is best to determine the actual voltage ratio by implementing a voltage divider with the potentiometer using the +15V and –15V sources and wiper.

For the dial marks to coordinate with the potentiometer wiper motion (i.e.: turning the dial toward increasing numbers results in smaller resistance between the higher potential terminal and the wiper, and therefore a higher voltage out of the voltage divider at the wiper terminal), **ALWAYS** connect the black plug to the lowest potential, ground or -15V, depending on the application.
Position Indicator

The Position Indicator unit consists of a 10KΩ potentiometer and a visual angular indicator on the front plate. The output Position Indicator is coupled directly to the Tachometer unit through a shaft. The top connection plate and the rotating front dial are shown in Figure 2.

![Position Indicator Diagram]

Figure 2 - Output Position Indicator Faceplate and Front Dial

The faceplate shows the connections to the 10KΩ potentiometer. The faceplate will be used for determining the coefficients of the Position Indicator, and the non-dynamic coefficients of the motor and generator.

The ends of the potentiometer are typically connected to +15V (plug 1) and -15V (plug 2). The potentiometer wiper (plug 3) is the output signal for indication of angular position, and is also used as a position feedback signal to the system.

The front plate has three sets of marks. The outermost indicates angular rotation in 10° increments. The inner two are used to synchronize angular velocity to either 50Hz (Europe) or 60Hz (USA) lighting systems.

The 60Hz ring has marks as seen on phonograph turntables. Room lighting provides an appropriate strobe effect, which is faster than the human eye can detect. The motor, by a gear reduction turns the output front plate. As the angular velocity is increased, the strobe effect is seen since the 60Hz portion of the plate appears to be stationary even though the plate is actually rotating. At the first appearance of this effect, the front plate is rotating at one revolution per second. Increasing the motor speed will bring a second strobe effect at two revolutions per second.

The Tachometer unit has a built-in digital display on the high-speed shaft section; the shaft speed can be directly read from the Tachometer unit.
Operational Amplifier

The operational amplifier is well known in Circuits courses and will only be briefly described here. The faceplate connections are shown in Figure 3.

![Operational Amplifier Faceplate](image)

At the top of the Op-Amp unit are three power connections for +15V, -15V, and ground. This D.C. power can be provided either from the Servo-Amp or the Power Supply.

As with the Pre-Amp, the Op-Amp has a Zero set dial, which must separately adjusted from the Pre-Amp. At the start of every lab session the uses the Op-Amp unit, the Zero Op-Amp Output Procedure (refer to Procedure section, page 43) must be performed to zero the output signal of the Op-Amp unit.

The Feedback Network Selector is a three-position rotary switch to select the feedback circuit for the Op-Amp. The bottom (counter-clockwise) switch selection has a 100KΩ resistor so that the Op-Amp unit, as shown in Figure 3, is a signal summer with a unity gain. The middle switch selection turns the Op-Amp circuit into a first order lag with $\tau = RC = 100\text{K}\Omega \cdot 1\mu\text{F} = 0.1 \text{ sec}$. The top (clockwise) switch selection allows for the insertion of any desired feedback network.
Lab Exercise 1 – Parameter Determination

**Purpose:**
In this lab exercise, the student will become familiar with the laboratory equipment and will also determine the various servo system parameters that will be used for the remainder of the lab exercises. Since the various components will be tested separately, the order of investigation is not critical. The suggested order for report presentation and lab investigation is:

1. Motor Generator Assembly:
   a. Motor Linearity
   b. Motor Dynamics
2. Position Indicator
3. Operational Amplifier Performance

![Figure 4 - "Core" System Plant Block Diagram](image)

![Figure 5 - “Core” System Plant Wiring Diagram](image)
Objectives:
After completing this exercise, you will be able to:
- Setup the “core” subject system plant: Pre-Amp, Servo-Amp, Motor, and Tachometer
- Perform the Balance Pre-Amp Output Procedure
- Measure RPM and D.C. voltages using the digital read-out on the Tachometer unit
- Measure the Phase Delay Angle between two sinusoidal wave-forms
- Measure the Time Constant of a first-order system step response

Also, the following system parameters will be measured:
- Maximum $V_{\text{Motor}}$ input for linear motor operation: $V_{\text{MAX}}$
- Motor Gain Constant: $K_m$ (RPM/Volt)
- Motor Time Constant: $\tau_m$ (seconds)
- Tachometer Gain Constant: $K_t$ (Volt/RPM)
- Position Indicator Gain Constant: $K_p$ (Volt/degree)
- Op-Amp Lag Feedback Time Constant: $\tau_1$ (seconds)

Reference Reading:
Model of a D.C. Motor – the derivation of the model of a D.C. motor is included in many Control Systems textbooks:
[1] Schaum’s Outline of Theory and Problems of Feedback and Control Systems, 2/e, Chapter 6, p 143
[5] Modern Control Engineering, 4/e, Chapter 3, pp139-141

Equipment List:
The following pieces of lab equipment will be required to complete this exercise:
- Pre-Amp, Servo-Amp, Motor, and Tachometer “core”
- Position Indicator
- Dual Attenuator
- Operational Amplifier
- Signal Generator
- Oscilloscope
- Printer
- Three coaxial cables with BNC to clip (alligator or microprobe)
- Various interconnect wires

Background Information:
The “plant” (object of control system) to be used in the lab exercises is a D.C., constant field, armature-current controlled motor. A second order differential equation describes the motor dynamics relating applied voltage to angular velocity. One system dynamic is due to the electrical effects of the armature coil inductance and resistance. The other dynamic system is due to the mechanical effects of the armature and load inertia and the bearing
friction. The electrical dynamics are much faster than mechanical dynamics. Thus, the motor can be modeled simply as a first order differential equation. The Servo-Amp and the Pre Amp are used to linearize the motor performance. The generator is physically much smaller than the motor and its dynamics do not influence the motor operation. The Motor Generator Assembly is represented by the block diagram in Figure 4, and its wiring diagram is shown in Figure 5.

Pre-Lab:
There is no pre-lab for this exercise.

Procedure:

Preliminary Preparation
Step 1 – Perform the Balance Pre-Amp Output Procedure (refer to Procedure section, page 31)

Maximum \( V_{Motor} \) Input: \( V_{MAX} \)
Note that the change in motor speed due to changes in \( V_{Motor} \) is not instantaneous. Therefore, allow several seconds between changing \( V_{Motor} \) and observing the corresponding motor speed.

Step 1 – Connect the Pre-Amp, Servo-Amp, Motor, and Tachometer as shown in Figure 5:
- The Dual-Attenuator provides a variable D.C. voltage, \( V_{Motor} \), to control motor speed. The motor speed should be zero near dial setting 5. The speed should increase in one direction as the dial is turned toward 10, and in the other direction as the dial is turned toward 0.
- On the Tachometer unit, connect the negative output from the Tachometer (plug 1) to the “tacho rpm” input (plug 3), and set the selector switch to read-out RPM.

Step 2 – Find the saturation \( V_{Motor} \) input – the point at which increasing \( V_{Motor} \) input no longer results in increasing motor speed. (Note: saturation is one of the non-linear behaviors that our model for the D.C. motor is ignoring.)
- Increase the \( V_{Motor} \) input by turning the dial from 5 toward 10 until the motor speed no longer increases (i.e.: the motor speed “saturates”).
- Slowly reduce \( V_{Motor} \) until the motor speed begins to decrease; this is the threshold \( V_{Motor} \) input for unsaturated motor operation – \( V_{SAT} \).
- Measure the voltage at the wiper of the Dual-Attenuator, \( V_{Motor} \), using the voltmeter read-out on the Tachometer unit.
- Record this value in the table below.

Step 3 – Compute the maximum \( V_{Motor} \) input for linear motor operation, \( V_{MAX} \):
- Compute \( V_{MAX} \): round 90% of \( V_{SAT} \) to the nearest 1/10 volt.
- Record this value in the table below.
- Note that \( \pm V_{MAX} \) should be the limits of \( V_{Motor} \) inputs in this lab exercise to avoid the nonlinear effects of motor speed saturation.
Motor Linearity: $K_m$ and $K_i$

The coefficients $K_m$ and $K_i$ will be determined by two methods.

Method 1 – Steady-State Operation

Step 1 – With the Dual Attenuator providing the D.C. $V_{Motor}$ input as shown in Figure 5 (setup in the previous procedure), adjust $V_{Motor}$ to fill in the following data table - $V_{Motor}$ vs. $V_{Tach}$:

- Use the volt meter read-out on the Tachometer unit to measure both $V_{Motor}$ and $V_{Tach}$.
- Use the RPM read-out on the Tachometer unit to measure the motor speed, $\omega_{HS}$.

Note: when setting inputs or other adjustable parameters to specified values, set the value as close as reasonably possible to the desired value (Input, [Approx.]), and then record the actual value (Set, [Actual]) rather than trying to get the adjustable value to be exact.

<table>
<thead>
<tr>
<th>Input: $V_{Motor}$ [Approx.]</th>
<th>Set: $V_{Motor}$ [Actual]</th>
<th>Measure: $V_{Tach}$</th>
<th>Motor Speed ($\omega_{HS}$)</th>
<th>Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{MAX}$</td>
<td>Volts</td>
<td>Volts</td>
<td>RPM</td>
<td>CW, CCW</td>
</tr>
<tr>
<td>$V_{MAX}/2$</td>
<td>Volts</td>
<td>Volts</td>
<td>RPM</td>
<td>CW, CCW</td>
</tr>
<tr>
<td>$-V_{MAX}$</td>
<td>Volts</td>
<td>Volts</td>
<td>RPM</td>
<td>CW, CCW</td>
</tr>
<tr>
<td>$-V_{MAX}/2$</td>
<td>Volts</td>
<td>Volts</td>
<td>RPM</td>
<td>CW, CCW</td>
</tr>
</tbody>
</table>

Table 1 - $V_{Motor}$ vs. $V_{Tach}$

Step 2 – Compute $K_m$ and $K_i$

Since the motor-tachometer assembly is assumed to be linear, the following ratios should be identical for all four table entries:

$$K_m = \frac{\Omega_{HS} (s)}{\Omega_{Motor} (s)} \frac{RPM}{Volts}$$

$$K_i = \frac{V_{Tach} (s)}{\Omega_{HS} (s)} \frac{Volts}{RPM}$$

Note: The upper-case variable for motor speed, $\Omega_{HS}$, in the equations above indicates that the equations are Transfer Functions in the Laplace domain. This is further indicated by the Laplace variable, $(s)$. The lower-case variable, $\omega_{HS}$, indicates a time-domain representation.
**Table 2 - \( K_m \) and \( K_t \)**

| \( V_{\text{Motor}} \) | \( K_m \) | \( K_t \) | \( |K_m| \) [Mean Deviation of Absolute Value] | \( |K_t| \) [Mean Deviation of Absolute Value] |
|------------------------|---------|---------|---------------------------------|---------------------------------|
| \( V_{\text{MAX}} \)  | RPM/Volt | Volt/RPM | %                              | %                              |
| \( V_{\text{MAX}}/2 \) | RPM/Volt | Volt/RPM | %                              | %                              |
| \( -V_{\text{MAX}} \)  | RPM/Volt | Volt/RPM | %                              | %                              |
| \( -V_{\text{MAX}}/2 \)| RPM/Volt | Volt/RPM | %                              | %                              |
| Mean of Absolute Values| RPM/Volt | Volt/RPM |                                 |                                 |

**Method 2 – Ramp Input**

Step 1 – Configure the system as shown in Figure 5 (setup in the previous procedure):

- For the \( V_{\text{Motor}} \) input to the Pre-Amp, **replace** the Dual Attenuator output with the function generator output.
- Set the function generator for Triangle-wave, 0.01 Hz, 3Vpp (limit to ±\( V_{\text{MAX}} \) Vpp)

Step 2 – Oscilloscope setup:

- Set the display mode to X-Y.
- Ground both Channel 1 (CH1) and Channel 2 (CH2) inputs.
- Center the beam dot vertically with the CH1 offset adjustment, and center it horizontally with either the CH2 offset adjustment or the time base offset adjustment (depending on the oscilloscope).
- Connect the oscilloscope CH1 input to \( V_{\text{Motor}} \).
  - Set CH1 (vertical displacement, Y-axis) scale to 0.5V/division.
- Connect the oscilloscope CH2 input to \( V_{\text{Tach}} \).
  - Set CH2 (horizontal displacement, X-axis) scale to 5V/division and **Invert** CH2.
- Set horizontal sweep to 10 seconds/div.
- Activate the Store feature of the oscilloscope and/or adjust the display persistence.

  The motor will turn; the speed will slowly increase, then slow, and reverse direction. This cycle will repeat due to the periodic Triangle-wave input. The trace on the scope will appear as a comet with an increasing tail. Eventually, the trace will look like a diagonal line on the screen.

Step 3 – Measure \( V_{\text{Motor}} \) and \( V_{\text{Tach}} \):

- Use the HOLD or RUN/STOP feature on the oscilloscope to capture the trace.
- Use the Hard Copy feature on the oscilloscope to obtain a print-out of the trace.
- Measure the **vertical extent** of the diagonal line; this is \( V_{\text{Motor}} \). Refer to CH1 setting for V/division scale.
• Measure the **horizontal extent** of the diagonal line; this is $V_{Tach}$. Refer to CH2 setting for V/division scale.
• Record the values of $V_{Motor}$ and $V_{Tach}$ in the table below.
• Indicate these measured ranges of $V_{Motor}$ and $V_{Tach}$ on the hard copy/print-out of the trace.

<table>
<thead>
<tr>
<th>$V_{Motor}$</th>
<th>Volts</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{Tach}$</td>
<td>Volts</td>
</tr>
</tbody>
</table>

Step 4 – Compare $V_{Motor}$ and $V_{Tach}$ with $K_m$ and $K_t$:
• The following ratio should hold:

$$\frac{V_{Tach}(s)}{V_{Motor}(s)} = K_m \cdot K_t$$

<table>
<thead>
<tr>
<th>Compute:</th>
<th>Analyze:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{V_{Tach}}{V_{Motor}}$</td>
<td>$K_m \cdot K_t$</td>
</tr>
<tr>
<td>%</td>
<td>%</td>
</tr>
</tbody>
</table>

• On the hard copy/print-out of the trace, indicate the location of the four data points $(V_{Motor}, V_{Tach})$ from Table 1.

**Motor Dynamics: $\tau_m$**

To determine the motor time constant, $\tau_m$, a faster time varying signal will be used as the input. Two methods will be used. The first method is a sinusoidal steady-state analysis, and the second method is a step-input response.

**Method 1 – Sinusoidal Steady-State Analysis**

Step 1 – Configure the system as shown in Figure 5, and with the **function generator** supplying the $V_{Motor}$ input to the Pre-Amp (setup in the previous procedure).
• Set the function generator for Sine-wave, 1 Hz, 2Vpp (limit to $\pm V_{MAX}$ Vpp)

Step 2 – Oscilloscope setup:
Change from previous procedure setup:
• Release the HOLD or RUN/STOP feature on the oscilloscope.
• Deactivate the Store feature and/or reduce the display.
• Set the display mode back to time-base (turn off X-Y display mode).

Step 3 – Measure Input/Output Waveform Phase Delay Angle:
• Measure the Phase Delay Angle, $\Theta$ - Perform the **Measure Sine-Waveform Phase Delay Angle Procedure** (refer to Procedure section, page 32).
  **Note:** the value of $\Theta$ should be negative since the motor transfer function has only Poles and no Zeros (i.e.: complex values only in the denominator) – and since the Output waveform ($V_{Tach}$) “Lags” the Input waveform ($V_{Motor}$).
• Use the Hard Copy feature on the oscilloscope to obtain a print-out of the traces.
• Record the values of $t_1$, $t_2$, and $\Theta$ in the table below.
• On the hard copy/print-out of the traces, indicate \( t_1 \) and \( t_2 \).

| \( t_1 \) | seconds |
| \( t_2 \) | seconds |
| \( \Theta \) | degrees |

Step 4 – Compute \( \tau_m \):

Discussion:

In general, the phase delay (phase shift) angle of a complex function is:

\[
\text{Phase Delay (degrees)} = \angle G(j\omega) = \tan^{-1}\left(\frac{\text{Im}\{G(j\omega)\}}{\text{Re}\{G(j\omega)\}}\right) \text{ degrees}
\]

For complex functions with numerators and denominators, the overall phase delay is computed as:

\[\text{Phase Delay} = (\text{Phase Delay of Numerator}) - (\text{Phase Delay of Denominator}) \text{ degrees}\]

For the D.C. motor under investigation, the overall phase delay is computed as:

\[
\angle\left(\frac{K_m \cdot K_i}{\tau_m s + 1}\right) = \angle(\text{Num}) - \angle(\text{Den}) = \tan^{-1}\left(\frac{\text{Im}\{K_m \cdot K_i\}}{\text{Re}\{K_m \cdot K_i\}}\right) - \tan^{-1}\left(\frac{\text{Im}\{\tau_m \cdot j\omega + 1\}}{\text{Re}\{\tau_m \cdot j\omega + 1\}}\right) \text{ degrees}
\]

\[
= (0) - \tan^{-1}\left(\frac{\tau_m \cdot \omega}{1}\right) \text{ degrees}
\]

• Compute the value of \( \tau_m \) using the Phase Delay Angle, \( \Theta \), measured in Step 3, and the relationship given above:

\[
\text{Phase Delay Angle, } \Theta = -\left(\frac{t_2}{t_1}\right) \cdot 360^\circ = -\tan^{-1}(\tau_m \cdot \omega) \text{ degrees}
\]

Noting that \( \omega = 2 \cdot \pi \cdot f = 2 \cdot \pi \cdot \frac{1}{T} = 2 \cdot \pi \cdot \frac{1}{t_i} \), the only unknown in the equation above is \( \tau_m \), which can be solved for using algebraic manipulation.

• Record the value of \( \tau_m \) below:

| \( \tau_m \) | seconds |

Method 2 – System Step-Input Response

Step 1 – Configure the system as shown in Figure 5, and with the function generator supplying the \( V_{\text{Motor}} \) input to the Pre-Amp (setup in the previous procedure):

• Set the function generator for Square-wave, 0.2 Hz, 3Vpp (limit to \( \pm V_{\text{MAX}} \) Vpp). The input waveform period must be slow enough to allow the system response from each step-input (i.e.: each edge of the Square-wave) to achieve a steady-state angular velocity before the next step-input occurs.
Step 2 – Oscilloscope setup:
Change from previous procedure setup:
• Release the HOLD or RUN/STOP feature on the oscilloscope.

Step 3 – Measure the Time Constant of the Motor, $\tau_m$:
• Perform the Measure Time Constant of a Step Response Procedure (refer to Procedure section, page 34).
• Use the Hard Copy feature on the oscilloscope to obtain a print-out of the step response.
• Record the measured value of $\tau_m$ below:

<table>
<thead>
<tr>
<th>$\tau_m$</th>
<th>seconds</th>
</tr>
</thead>
</table>

• On the hard copy/print-out of step response, indicate $\tau_m$.

Step 4 – Analyze/Compare the two values of $\tau_m$ from Method 1 – Sinusoidal Steady-State Analysis – and from Method 2 – System Step-Input Response:

<table>
<thead>
<tr>
<th>Summary:</th>
<th>Analyze:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_m$</td>
<td>$\tau_m$ [Mean Deviation]</td>
</tr>
<tr>
<td>$\tau_m$ - Method 1</td>
<td>seconds</td>
</tr>
<tr>
<td>$\tau_m$ - Method 2</td>
<td>seconds</td>
</tr>
<tr>
<td>Mean</td>
<td>seconds</td>
</tr>
</tbody>
</table>

**Position Indicator: $K_p$**
The Position Indicator is the potentiometer assembly that is connected to the motor via the gearbox. Its transfer function is described by a constant, $K_p$, that relates angular displacement to voltage.

Step 1 – Disconnect the motor by removing the pair of connections between the Pre-Amp and the Servo-Amp (refer to Figure 5). This will allow for free motion of the motor by hand.

Step 2 – Setup for the Position Indicator:
• Connect +15V and -15V sources to the Position Indicator as shown in Figure 2, with +15V on plug 1 and -15V on plug 2.
• Connect the potentiometer wiper (plug 3), $V_{position}$, to the digital volt meter read-out on the Tachometer unit.

Step 3 – Measure and record voltages, $V_{position}$, at various angular positions:
• Manually rotate the motor high-speed shaft ($\omega_{HS}$ in Figure 5) – do NOT rotate the potentiometer face-plate – and measure and record the values in Table 3. For
a sign convention, consider Clockwise angles as positive and Counter-Clockwise angles as negative. (The actual sign is not important; the overall change in voltage per change in angle is being measured.)

- For each pair of angle and voltage measurements, compute the change in voltage with respect to angular position, $\Delta$Volts/$\Delta$degrees. (Consider only the absolute value; disregard any sign.)

- Compute the mean value of $\Delta$Volts/$\Delta$degrees.

<table>
<thead>
<tr>
<th>Set: $\Theta_{\text{Motor}}$ [Approx.]</th>
<th>Measure: $V_{\text{Position}}$</th>
<th>Compute: $\Delta$Volts / $\Delta$degrees [Absolute Value]</th>
<th>Analyze: Volt/degree [Mean Deviation]</th>
<th>Volt/degree [Range Deviation]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-60°</td>
<td>Volts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-50°</td>
<td>Volts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-40°</td>
<td>Volts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-30°</td>
<td>Volts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-20°</td>
<td>Volts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-10°</td>
<td>Volts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0°</td>
<td>Volts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10°</td>
<td>Volts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20°</td>
<td>Volts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30°</td>
<td>Volts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40°</td>
<td>Volts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50°</td>
<td>Volts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60°</td>
<td>Volts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean: $K_p$</td>
<td>Volts</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3 - Position Indicator Angular Position and Voltage

**Operational Amplifier Performance: Gain Verification and $\tau_1$**

The Operational Amplifier (Op-Amp) unit will be used in later lab exercises as a summing junction, and to add dynamics (i.e.: poles) to the overall system.

**Step 1 – Setup for the Op-Amp:**
- Connect the three power connections (+15V, 15V, and ground) on the Op-Amp.
- Connect one of the three Op-Amp input plugs (1, 2, or 3) to ground.
- Set the three-position **Feedback Network Selector** rotary switch to the 100KΩ resistor.
- Use the volt meter read-out on the Tachometer unit to measure the output voltage of the Op-Amp.
• Adjust the **Zero Set** dial on the Op-Amp unit until the output voltage (plug 6) shows 0 Volts.
• Remove the ground input to the Op-Amp.

**Step 2 – Check the Op-Amp Gain and Summing function:**
• Use both potentiometers on the Dual-Attenuator unit as voltage dividers. Set one of them to provide +3V at the wiper terminal, and set the other to provide -5V at the wiper terminal. (Get the values as close as reasonably possible and then record the actual values.)
• Connect the +3V source to one input of the Op-Amp (plug 1, 2, or 3), and the -5V source to one of the other inputs.
• The Op-Amp configuration is an inverting summer. Therefore, the output voltage should be:

\[
- (V_1 + V_2) = -(3V + -5V) = 2V
\]

Measure the actual Op-Amp output voltage and compare it with the expected value (using your measured input voltages to determine the expected value).

<table>
<thead>
<tr>
<th>Input:</th>
<th>Set:</th>
<th>Compute:</th>
<th>Measure:</th>
<th>Analyze:</th>
</tr>
</thead>
<tbody>
<tr>
<td>V_1</td>
<td>V_2</td>
<td>V_1 [Actual]</td>
<td>V_2 [Actual]</td>
<td>V_{Out [Expected]}</td>
</tr>
<tr>
<td>3V</td>
<td>-5V</td>
<td>Volts</td>
<td>Volts</td>
<td>Volts</td>
</tr>
</tbody>
</table>

**Step 3 – Measure the Op-Amp Resistor/Capacitor Feedback Network Time Constant, \( \tau_1 \):**
• Remove the +3V and -5V inputs to the Op-Amp.
• Set the three-position **Feedback Network Selector** rotary switch to the 1 µF capacitor in parallel with the 100KΩ resistor.
• Set the function generator for Square-wave, 0.2 Hz, 5Vpp.
• Connect the output of the function generator to one of the Op-Amp inputs.
• Connect the Op-Amp output to one of the oscilloscope input channels.
• Perform the **Measure Time Constant of a Step Response Procedure** (refer to Procedure section, page 34). Measure and record the time constant, \( \tau_1 \).
• Use the Hard Copy feature on the oscilloscope to obtain a print-out of the step response.
• On the hard copy/print-out of step response, indicate \( \tau_1 \).

<table>
<thead>
<tr>
<th>Expected:</th>
<th>Measured:</th>
<th>Analyze:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_1 ) [Expected]</td>
<td>( \tau_1 ) [Actual]</td>
<td>% Error</td>
</tr>
<tr>
<td>0.1 seconds</td>
<td>seconds</td>
<td>%</td>
</tr>
</tbody>
</table>

• In the Lab Report, derive the transfer function of the Op-Amp with the R-C feedback network, and verify that the expected characteristic behavior is indeed a first-order exponential with a time constant of 0.1 seconds.
Summary:
Lab Exercise Checklist – Be sure that the following have been obtained in the Lab to complete the lab exercise:

- Collected data supporting the measurements of $V_{\text{MAX}}$, $K_m$, $\tau_m$, $K_t$, $K_p$, and $\tau_1$
- Annotated hardcopies of traces from Motor Linearity, from both methods of measuring $\tau_m$ and from measuring $\tau_1$

Lab Report Checklist – The Lab Report should contain the following supporting documentation:

- All collected data and hardcopies of scope traces
- Parameter values (to be used in the remainder of lab exercises). [Where the value was measured by more than one method, or was taken more than one time, use the mean value.]
- Error or appropriate deviation analysis of the measured data
- Derivation of Op-Amp transfer function with R-C feedback network

Summary of Measured Parameter Values – to be used in the remainder of the lab exercises:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{\text{MAX}}$</td>
<td>Volts</td>
</tr>
<tr>
<td>$K_m$</td>
<td>RPM/Volt</td>
</tr>
<tr>
<td>$\tau_m$</td>
<td>seconds</td>
</tr>
<tr>
<td>$K_t$</td>
<td>Volt/RPM</td>
</tr>
<tr>
<td>$K_p$</td>
<td>Volt/degree</td>
</tr>
<tr>
<td>$\tau_1$</td>
<td>seconds</td>
</tr>
</tbody>
</table>

The following should be considered as reference values of the system parameters for "sanity check" purposes only. They should NOT be considered as "correct" or "theoretical" values for error analysis.

- Motor Gain Constant – $K_m$: 1800-2600 RPM/Volt
- Motor Time Constant – $\tau_m$: 0.20-0.26 seconds
- Tachometer Gain Constant – $K_t$: 0.0023-0.0027 Volt/RPM
- Position Indicator Gain Constant – $K_p$: 0.07-0.12 Volt/Degree
- Op-Amp Lag Feedback Time Constant – $\tau_1$: 0.09-0.11 seconds

Review Questions – Answers must be included in the lab report:

- Is it justifiable to model the motor as a first-order system, considering only the mechanical pole and ignoring the electrical pole from the inductance of the armature (as well as ignoring nonlinear effects of motor saturation and gear backlash, and armature resistance changes with temperature)? Why or why not? (Consider your deviation and difference analyses of the motor parameters $K_m$ and $\tau_m$.)
Procedure: Balance Pre-Amp Output

Purpose:
This procedure must be performed before each lab exercise to balance the differential output signal of the Pre-Amp unit.

Procedure:
Step 1 – Connect the Pre-Amp, Servo-Amp, Motor, and Tachometer as shown in Figure 6.
  • Connect the three power connections (+15V, 15V, and ground) on both the Pre-Amp and Tachometer units from either the Servo-Amp or the Power Supply.
  • On the Pre-Amp unit, set the three-position switch to “defined $\tau$.”

Step 2 – Ground the “V-Motor” input (plug 1) on the Pre-Amp unit. This provides a “command” input for Zero speed from the motor.

Step 3 – Adjust the Zero Set dial on the Pre-Amp unit until the motor does not turn.
**Procedure: Measure Sine-Waveform Phase Delay Angle**

**Purpose:**
This procedure is used to measure the Phase Delay Angle (in degrees) between two sinusoidal wave-forms.

![Figure 7 – CH1 and CH2 Period and Phase Delay Time](image)

**Procedure:**
Step 1 – Oscilloscope setup:
- Connect the oscilloscope CH1 input to the system “input” signal (e.g.: $V_{Motor}$).
- Connect the oscilloscope CH2 input to system “output” signal (e.g.: $V_{Tach}$).
- Invert CH2.
- Adjust the CH1 and CH2 vertical sensitivities (Volt/division) so that both Sine-waves have about the same amplitude.
- Adjust horizontal sweep (seconds/division) until a complete Sine-wave (from Peak to Peak) is displayed on the screen from both CH1 and CH2.
Step 2 – Measure Waveform Period and Phase Delay Time:
- Use the HOLD or RUN/STOP feature on the oscilloscope to capture the trace.
- Use the Cursor or Measure feature on the oscilloscope to measure the **Period** of the CH1 signal – the time between one peak and the next peak of the waveform; this is $t_1$. (Refer to Figure 7.)
- Use the Cursor or Measure feature on the oscilloscope to measure the **Phase Delay Time** between the CH1 and CH2 signals – the time between the peak of the leading waveform and the corresponding peak of the lagging waveform; this is $t_2$. (Refer to Figure 7.)

Step 3 – Compute the Waveform Phase Delay Angle:
- In general, the phase delay (phase shift) angle between two sinusoidal function is the ratio of the time delay between the two functions ($t_2$) and the period ($t_1$) expressed as a fraction of one complete cycle ($360^\circ$). This is computed as:

\[
\text{Phase Delay (degrees), } \Theta = \left(\frac{t_2}{t_1}\right) \cdot 360^\circ
\]

- For complex functions with complex numerators and denominators, the overall phase delay computed as:

\[
\text{Phase Delay (degrees), } \Theta = (\text{Phase Delay of Numerator}) - (\text{Phase Delay of Denominator}) \, \text{degrees}
\]

- The two phase delay calculations should be equal. Example: for the D.C. motor under investigation, the overall phase delay is computed as:

\[
\angle \left( \frac{K_m \cdot K_i}{\tau_m s + 1} \right) s = j \omega = \angle(\text{Num}) - \angle(\text{Den}) = (0) \cdot \left(\frac{t_2}{t_1}\right) \cdot 360^\circ
\]
Procedure: Measure Time Constant of a Step Response

Purpose:
This procedure is used to measure the Time Constant from a first-order exponential system step response.

Procedure:
Step 1 – Oscilloscope setup:
- Connect one oscilloscope input channel to the signal being evaluated (e.g.: VTach).
- Adjust the vertical sensitivity (Volt/division) so that the entire amplitude of the step response fits on the screen.
- Adjust horizontal sweep (seconds/division) so that a single step response (from the beginning of the step until the final value is reached) is displayed on the screen.

Step 2 – Measure the Exponential Time Constant:
- Use the HOLD or RUN/STOP feature on the oscilloscope to capture a positive going step response.
- Use the Cursor or Measure feature on the oscilloscope to measure the overall vertical displacement of the step response (from the beginning of the step to the final value – Refer to Figure 7.) Call this 100%.

The Time Constant of an exponential decay curve is the time required to reach 37% ($e^{-1}$) of the initial value.

![Figure 8 – Measuring the Time Constant from a First-Order Exponential Step Response](image)
The Time Constant of an exponential rise curve is the time required to reach 63% 
\((1-e^{-1})\) of the final value.

- Compute 63% of the overall vertical displacement of the rising step response.

The horizontal cursor will be used to measure the time from the beginning of the step 
response to the 63% point, however vertical cursor indicating the 63% vertical location 
will be lost when the cursor mode is switched from vertical to horizontal.

- Use the horizontal offset adjustment on the oscilloscope to align the intersection of 
the trace of the step response and the 63% cursor indication with one of the solid 
vertical lines on the oscilloscope display. (Refer to the “Triple Intersection” in 
Figure 8.)

- Switch the cursor measurement mode from vertical to horizontal, and measure the 
time from the beginning of the step response to the 63% point (the intersection of 
the trace of the step response with the solid vertical line on the oscilloscope 
display).

- This time measurement is the time constant, \(\tau\), of the exponential curve.
Lab Exercise 2 – Closed-Loop Position Control System Design

Purpose:
The purpose of this lab exercise is to design and evaluate a closed-loop position control system, predict its behavior, and control its behavior. The component parameters measured in Lab Exercise #1 will be used to model the system components.

Figure 9 - Position Control Loop Block Diagram

Figure 10 - Position Control Loop Wiring Diagram
Objectives:
After completing this exercise, you will be able to:
- Perform the Zero Op-Amp Output Procedure
- Predict second-order system response to a step-input
- Design basic second-order system step response by adjusting gain
- Measure the Percent Overshoot, Peak-Time, and Settling-Time of an under-damped second-order step response

Reference Reading:
   Block-Diagram Simplification:
   - Chapter 7, pp 154-207
   Block-Diagram Simplification:
   - Chapter 2, pp 62-71
   Second-Order Dynamic Response Design Procedure & Equations:
   - Chapter 5, pp 227-232
   - Summary on inside back cover
   Block-Diagram Simplification:
   - Chapter 3, pp 123-133
   Second-Order Dynamic Response Design Procedure & Equations:
   - Chapter 3, pp 136-150
   - Summary on p. 180 and inside back cover.
   Block-Diagram Simplification:
   - Chapter 5, pp 252-263
   Second-Order Dynamic Response Design Procedure & Equations:
   - Chapter 4, pp 191-202
   - Summary on inside back cover.
   Block-Diagram Simplification:
   - Chapter 3, pp 58-61
   Second-Order Dynamic Response Design Procedure & Equations:
   - Chapter 5, pp 224-235

Equipment List:
The following pieces of lab equipment will be required to complete this exercise:
- Pre-Amp, Servo-Amp, Motor, and Tachometer “core”
- Position Indicator
- Dual Attenuator
- Operational Amplifier
- Signal Generator
- Oscilloscope
- Printer
- Three coaxial cables with BNC to clip (alligator or microprobe)
Various interconnect wires

**Background Information:**
Lab Exercise #1 examined the open-loop motor. The transfer function for the motor that relates applied voltage to motor speed is a first-order function; the motor’s changes in speed due to a square-wave input exhibited the characteristic exponential step response of a first-order system. The new system transfer function introduces a 1/s term (discussed later), which results in an overdamped second-order system. However, as the system gain, $K_a$, is adjusted, the closed-loop system poles move from their open-loop positions; the system becomes critically-damped at some value of $K_a$, and is under-damped for $K_a$ greater than that value. The objectives of this lab exercise include predicting the characteristics of the under-damped step response with a given value of $K_a$, and specifying the value of $K_a$ to achieve the specified under-damped step response characteristics.

The Closed-Loop Position Control system is represented by the block diagram in Figure 9, and its wiring diagram is shown in Figure 10.

The new components that are introduced in the block diagram are the summing junction, the variable gain, $K_a$, the gear reduction factor, 1/30, the unit conversion factor, 6, and the integrator, 1/s.

The summing junction is implemented with the Op-Amp using the resistor feedback network for a unity gain (gain factor of 1). (Note that the reversal of direction from the gear coupling provides the negative sign on the summing junction’s feedback input.)

The variable gain, $K_a$, is implemented with one of the attenuators (potentiometers) configured as a voltage divider. Since the gain is implemented with an attenuator, the attainable gain is limited to values between zero and one.

The gear reduction factor is just the transfer function of the gear-train:

$$
\frac{Output(s)}{Input(s)} = \frac{\# \text{revolutions on Output shaft}}{\# \text{revolutions on Input shaft}} = \frac{1}{30}
$$

Note that this transfer function ignores friction, the mass of the gears, gear backlash, etc. It would also be possible, if desired, to derive transfer functions for the gear-train that relate other input-output relationships such as input torque vs. output torque.

The unit conversion factor, 6, appears in the block diagram but is not a physical element in the system. Since the digital read-out on the Tachometer unit displays motor angular speed as RPM (Revolutions-per-minute), and the position indicator produces a voltage relative to degrees (rather than revolutions), a conversion factor is necessary for consistency of units:

$$
\text{Conversion Factor} : 6 \left( \frac{\text{minute} \cdot \text{degree}}{\text{second} \cdot \text{revolution}} \right)
$$
The Integrator (1/s), like the unit conversion factor, is not a physical element in the system. Some transfer function derivations for the D.C. motor include the 1/s term, and relate applied voltage to angular position (recall that the motor transfer function used here relates applied voltage to angular velocity). By definition of the motor transfer function, the quantities on the High-Speed and Low-Speed shafts are velocities, $\omega_{HS}$ and $\omega_{LS}$. Since the output of the Position Indicator is a voltage related to position, $V_{Position}$, then conceptually integration (1/s) is occurring based only on our definitions of the quantities, and not though a physical element performing an integration function.

**Pre-Lab:**
Step 1 – Derive the Closed-Loop Transfer Function:
- Derive the closed-loop transfer function, $T(s) = V_p(s)/V_{In}(s)$, for the system shown in Figure 9. The transfer function should be derived with the symbols in the block diagram ($K_a, K_m, \tau_m, K_p$), and then the values measured in Lab Exercise #1 should be substituted in. Note that the gain, $K_a$, is still a variable.

Additionally, it may be convenient to algebraically manipulate the transfer function so that the coefficient of the $s^2$ term in the denominator is 1.

Step 2 – Predict Transient Response for $K_a = 1$:
- **Predict** the percent overshoot, $M_p$, peak time, $t_p$, and settling time, $t_s$, for the system when $K_a = 1$. Set $K_a = 1$ in the transfer function derived in Step 1 of the pre-lab, and use the second-order system design equations (from class lecture, the lab instructor, or from a control systems textbook – see references) to determine the expected $M_p$, $t_p$, and $t_s$ for the system step-response.

<table>
<thead>
<tr>
<th>Expected for $K_a = 1$:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_p$</td>
<td>seconds</td>
</tr>
<tr>
<td>$t_s$</td>
<td>seconds</td>
</tr>
<tr>
<td>$M_p$</td>
<td>%</td>
</tr>
</tbody>
</table>

Step 3 – Compute $K_a$ for Specified Transient Response:
- **Determine the value of $K_a$ that will yield a 25% overshoot** to a step input for the transfer function derived in Step 1 of the pre-lab, and use the second-order system design equations as needed.
- Also, **predict** the peak time, $t_p$, and settling time, $t_s$, at this value of $K_a$.

<table>
<thead>
<tr>
<th>Expected for $M_p = 25%$:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_a$</td>
<td></td>
</tr>
<tr>
<td>$t_p$</td>
<td>seconds</td>
</tr>
<tr>
<td>$t_s$</td>
<td>seconds</td>
</tr>
</tbody>
</table>
Procedure:

Preliminary Preparation

Step 1 – Perform the Balance Pre-Amp Output Procedure (refer to Procedure section, page 31)
Step 2 – Perform the Zero Op-Amp Output Procedure (refer to Procedure section, page 43)

Step Response for Ka = 1

Step 1 – Connect the system components as shown in Figure 10:

- Set the gain, Ka, to 1.
The Dual-Attenuator provides the variable gain, K_a. With the black plug of the attenuator connected to ground, the gain will increase as the dial is turned toward 10 (voltage increases at the wiper terminal), and decrease as the dial is turned toward 0 (voltage decreases at the wiper terminal). Therefore, a gain of 1 should be observed with the dial turned all the way to 10. (This can be verified with the Set Gain using an Attenuator Procedure – page 44.)

Step 2 – Function Generator setup:

- Set the function generator for Square-wave output, frequency low enough so that the step response has sufficient settling time before the next edge of the square-wave (use the settling time, t_s, as a guide), amplitude for 50° peak-to-peak.
Use K_p to convert from the angular specification to voltage.
Note that each edge of the square-wave represents half of the period of the overall waveform.
- Connect the function generator output to the Op-Amp input, Vin, as shown in Figure 10.
- Confirm that the VMotor input to the servo-amp is limited to ±V_MAX Vpp; reduce Vin if necessary.

Step 3 – Oscilloscope setup:

- Connect the oscilloscope CH1 input to the Input Voltage, V_in.
  Adjust the vertical sensitivity (Volt/division) of CH1 so that the entire amplitude of the square-wave input fits on the screen.
- Connect the oscilloscope CH2 input to V_Position, the output of the Position Indicator (refer to Figure 10).
  Adjust the vertical sensitivity of CH2 so that the entire amplitude of the step response fits on the screen.
  Invert CH2.
- Adjust horizontal sweep (seconds/division) until a single step response (from the beginning of the step until the final value is reached) is displayed on the screen.

Step 4 – Measure and Record the Step Response characteristics:

- Perform the Measure Characteristics of an Under-Damped Step Response Procedure (refer to Procedure section, page 45).
- Use the Hard Copy feature on the oscilloscope to obtain a print-out of the step response.
• Record the measured values of $t_p$, $t_s$ and $M_p$ below and compare the measured values with the expected values computed in the second part of the pre-lab:

<table>
<thead>
<tr>
<th></th>
<th>Measured:</th>
<th>Analyze:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_p$</td>
<td>seconds</td>
<td>%</td>
</tr>
<tr>
<td>$t_s$</td>
<td>seconds</td>
<td>%</td>
</tr>
<tr>
<td>$M_p$</td>
<td>%</td>
<td>%</td>
</tr>
</tbody>
</table>

• On the hard copy/print-out of step response, indicate $t_p$, $t_s$ and $M_p$.

**Step Response for $M_p = 25\%$**

Step 1 – With the same system component connections as shown in Figure 10:

- Use the Set Gain using an Attenuator Procedure (refer to Procedure section, page 44) to set the gain, $K_a$, to the value computed in the third part of the pre-lab to achieve a 25% overshoot.

Step 2 – Function Generator setup – setup in the previous procedure:

- Set the function generator for Square-wave output, frequency low enough so that the step response has sufficient settling time before the next edge of the square-wave (use the settling time, $t_s$, as a guide), amplitude for 50° peak-to-peak. Use $K_p$ to convert from the angular specification to voltage. Note that each edge of the square-wave represents half of the period of the overall waveform.

- Connect the function generator output to the Op-Amp input, $V_{in}$, as shown in Figure 10.

- Confirm that the $V_{Motor}$ input to the servo-amp is limited to $\pm V_{MAX} V_{pp}$; reduce $V_{in}$ if necessary.

Step 3 – Oscilloscope setup – setup in the previous procedure:

- Release the HOLD or RUN/STOP feature on the oscilloscope (from the previous procedure).

- Connect the oscilloscope CH1 input to the Input Voltage, $V_{in}$. Adjust the vertical sensitivity (Volt/division) of CH1 so that the entire amplitude of the square-wave input fits on the screen.

- Connect the oscilloscope CH2 input to $V_{Position}$, the output of the Position Indicator (refer to Figure 10), adjust the vertical sensitivity of CH2 so that the entire amplitude of the step response fits on the screen. Invert CH2.

- Adjust horizontal sweep (seconds/division) until a single step response (from the beginning of the step until the final value is reached) is displayed on the screen.

Step 4 – Measure and Record the Step Response characteristics:

- Perform the Measure Characteristics of an Under-Damped Step Response Procedure (refer to Procedure section, page 45).
• Use the Hard Copy feature on the oscilloscope to obtain a print-out of the step response.
• Record the measured values of $t_p$, $t_s$ and $M_p$ below and compare the measured values with the expected values computed in the third part of the pre-lab:

<table>
<thead>
<tr>
<th>Measured:</th>
<th>Analyze:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_p$</td>
<td>seconds</td>
</tr>
<tr>
<td>$t_s$</td>
<td>seconds</td>
</tr>
<tr>
<td>$M_p$</td>
<td>%</td>
</tr>
</tbody>
</table>

• On the hard copy/print-out of step response, indicate $t_p$, $t_s$ and $M_p$.

**Summary:**
Lab Exercise Checklist – Be sure that the following have been obtained in the Lab to complete the lab exercise:
- Collected data supporting the measurements of $t_p$, $t_s$ and $M_p$ for both $K_a = 1$ and $M_p = 25$
- Annotated hardcopies of step response traces for both $K_a = 1$ and $M_p = 25$

Lab Report Checklist – The Lab Report should contain the following supporting documentation:
- All collected data and hardcopies of scope traces
- Derivation of closed-loop transfer function
- Pre-lab computations
- Error analysis of the measured vs. expected values

**Review Questions – Answers must be included in the lab report:**
• Are the second-order system design equations adequate to predict and design the response of the under-damped step response? Why or why not? (Consider your error analyses of the measured vs. expected values of $t_p$, $t_s$ and $M_p$.)

Procedure: Zero Op-Amp Output

Purpose:
This procedure must be performed before each lab exercise that uses the Op-Amp to zero the output signal of the Op-Amp unit.

Note: This procedure must be performed AFTER performing the Balance Pre-Amp Output Procedure.

![Figure 11 – Op-Amp Zero Output Adjust](image)

Procedure:
Step 1 – Connect the Op-Amp, Pre-Amp, Servo-Amp, Motor, and Tachometer as shown in Figure 11.
- Connect the three power connections (+15V, 15V, and ground) on the Op-Amp, Pre-Amp, and Tachometer units from either the Servo-Amp or the Power Supply.
- On the Op-Amp unit, set the three-position selector to the single resistor feedback.
- On the Pre-Amp unit, set the three-position switch to “defined $\tau$.”

Step 2 – Ground the “V-In” input (plug 1) on the Op-Amp unit. This provides a “command” input for Zero speed from the motor.

Step 3 – Adjust the Zero Set dial on the Op-Amp unit until the motor does not turn.
Procedure: Set Gain using an Attenuator

Purpose:
This procedure describes how to set a specified gain value between the values of zero and one using an attenuator (potentiometer).

\[ V_{out} = V_{in} \times K_a \]

Figure 12 – Setting Gain using an Attenuator

Procedure:
Step 1 – Disconnect the V_{out} (wiper) terminal on the attenuator from other “downstream” connections. This will prevent the V_{out} voltage from driving the motor.

Step 2 – Connect the +15V source voltage from either the Servo-Amp or the Power Supply as V_{in}, as shown in Figure 12.

Step 3 – Compute V_{out}.
- Measure and record V_{in} using the digital voltmeter read-out on the Tachometer unit. V_{in} is supposed to be +15V; this is just for confirmation.
- Compute V_{out} as the product of the measured value of V_{in} and the desired gain value.

Step 4 – Set V_{out}.
- Connect the V_{out} (wiper) terminal on the attenuator to the digital voltmeter input on the Tachometer unit (plug 4).
- Adjust the potentiometer dial until V_{out} equals (reasonably closely; within a few tenths of one volt) the computed V_{out} value from Step 3.

Step 5 – Reconnect the attenuator V_{in} and V_{out} terminals to their original connections.
**Procedure: Measure Characteristics of an Under-Damped Step Response**

**Purpose:**
This procedure is used to measure the Rise-Time, Peak-Time, Settling-Time, and Percent Overshoot of an under-damped second-order step response.

![Diagram of under-damped step response showing t_r, t_p, t_s, and peak overshoot.]

**Procedure:**
Step 1 – Oscilloscope setup:
- Connect one oscilloscope input channel to the signal being evaluated (e.g.: V_{position}).
- Adjust the vertical sensitivity (Volt/division) so that the entire amplitude of the step response fits on the screen (including the peak overshoot).
- Adjust horizontal sweep (seconds/division) so that a single step response (from the beginning of the step until the final value is reached) is displayed on the screen.
- Use the HOLD or RUN/STOP feature on the oscilloscope to capture a positive going step response.
Step 2 – Measure the Final Value:
- Use the Cursor or Measure feature on the oscilloscope to measure the overall vertical displacement of the step response from the beginning of the step to the final value – Refer to Figure 13. Call this 100%.

Note that due to disturbance inputs, system nonlinearities, etc., the oscillations may never decay to within the 2% (±1%) settling band. (This condition should be noted in your lab report.) If so, estimate and measure the value that the signal is oscillating around and use this as the final value.

If required:
Step 3 – Measure the Rise-Time, \( t_r \):
- Use the Cursor or Measure feature on the oscilloscope to measure the time that it takes for the step response to transition from 10% to 90% or the final value. (Refer to Figure 13.)

If required:
Step 4 – Measure the Peak-Time, \( t_p \):
- Use the Cursor or Measure feature on the oscilloscope to measure the time that it takes for the step response to transition from the beginning of the step response to the first peak of the oscillation. (Refer to Figure 13.)

If required:
Step 5 – Measure the Settling-Time, \( t_s \):
- Use the Cursor or Measure feature on the oscilloscope to measure the time that it takes for the step response to transition from the beginning of the step response until the oscillation remains within 2% (±1%) of the final value (i.e.: the settling band). (Refer to Figure 13.)

If the oscillations do not decay to within the 2% (±1%) settling band, this condition should be noted in your lab report.

If required:
Step 6 – Measure the Percent Overshoot, \( M_p \):
- Use the Cursor or Measure feature on the oscilloscope to measure the amount of overshoot of the first peak of the oscillation above the final value. (Refer to Figure 13.)
- Compute the percent overshoot as:

\[
Percent\ Overshoot, \ M_p = \frac{Overshoot}{Final\ Value} \times 100\% = \frac{Peak\ Value - Final\ Value}{Final\ Value} \times 100\%
\]
Lab Exercise 3 – Design for Steady-State Error and Marginal Stability

Purpose:
The purpose of this lab exercise is to design and evaluate a type-1 position control system’s ability to track a ramp-input to a specified steady-state error, and to design a third-order system to operate on the threshold of stability.

Objectives:
After completing this exercise, you will be able to:
- Predict a type-1 system’s expected steady-state error to a ramp input
- Design a systems steady-state error by adjusting gain
- Measure the system’s steady-state error to a ramp input
- Predict a marginally stable system’s oscillation frequency
- Design a system to operate on the threshold of stability by adjusting gain

Reference Reading:
- References from Previous Lab Exercises:
  - Block-Diagram Simplification
  - Second-Order Dynamic Response Design Procedure & Equations

  - Steady-State Error:
    - Chapter 9, pp 214-219
  - Stability:
    - Chapter 5, pp 114-127

  - Steady-State Error:
    - Chapter 5, pp 240-244
  - Stability:
    - Chapter 6, pp 290-300

  - Steady-State Error:
    - Chapter 4, pp 234-236
  - Stability:
    - Chapter 3, pp 157-166

  - Steady-State Error:
    - Chapter 7, pp 368-383
  - Stability:
    - Chapter 6, pp 324-347

Steady-State Error:
- Chapter 5, pp 288-293

Stability:
- Chapter 5, pp 275-281

Figure 14 - Position Control Loop Block Diagram indicating Error Signal

(Note: Wiring diagram is identical to Figure 10, page 36.)
Figure 15 - Third-Order Position Control System Block Diagram for Marginal Stability Design

Figure 16 - Third-Order Position Control System Wiring Diagram

**Equipment List:**
The following pieces of lab equipment will be required to complete this exercise:

- Pre-Amp, Servo-Amp, Motor, and Tachometer “core”
- Position Indicator
- Dual Attenuator
- Operational Amplifier
- Signal Generator
- Oscilloscope
- Printer
- Three coaxial cables with BNC to clip (alligator or microprobe)
- Various interconnect wires
Background Information:

Steady-State Error

In Lab Exercise #2, the system design criteria, $t_p$, $t_s$ and $M_p$, were based on the transient response with respect to the final value from a step input. Some design specifications consider how closely the steady-state output tracks the input signal. The Error signal, $E(s)$, is defined as the difference between the system input and the system output. While the system Transfer Function examines the ratio of the system output over the input in the frequency (“$s$”) domain, the system Steady-State Error, $e_{ss}$, examines the Difference (i.e.: Error) between the system input and output in the time domain.

Many control system textbooks examine system steady-state error with respect to the number of system poles at the origin (i.e.: $1/s$ terms – the number of system poles at the origin).

Note that the System Type, number of poles at the origin, is independent of the system order (number of poles in the system transfer function), other than the fact that the system order must be at least equal to the system type.

This lab exercise examines the ability of a type 1 system (one pole at the origin) to track a ramp input ($R/s^2$).

Marginal Stability

System stability examines whether a system’s output decays to a final value or remains bounded over time (stability), or grows toward infinity or to the maximum extent of the system output capability – such as an op-amp output getting “stuck on the rails” – (instability).

For a system to be stable, all system poles must have negative real parts (i.e.: be to the left of the $j\omega$-axis; “be in the Left-Half Plane”). A system is unstable if any pole has a positive real part (i.e.: lies to the right of the $j\omega$-axis; “is in the Right-Half Plane”).

A system’s output will oscillate if it has complex poles. If the complex poles are stable (lie in the Left-Half Plane), then the oscillation amplitude will exponentially decay; if the complex poles are unstable (lie in the Right-Half Plane), the oscillation amplitude will exponentially grow. When a system’s complex poles lie directly on the $j\omega$-axis, the oscillation is “un-damped”; the system output oscillates with constant amplitude. This condition is called marginal stability.

Pre-Lab:

Steady-State Error

Step 1 – Derive the Error Transfer Function:

- Derive the error transfer function, $T_E(s) = V_{error}(s)/V_{In}(s)$, for the system shown in Figure 14. The error transfer function should be derived with the symbols in the block diagram ($K_a$, $K_m$, $\tau_m$, $K_p$), and then the values measured in Lab Exercise #1 should be substituted in. Note that the gain, $K_a$, is still a variable.
Step 2 – Derive the System Error Function, $V_{error}(s)$, to a Ramp-Input, $V_{in}(s) = R/s^2$:

- $V_{error}(s) = V_{in}(s) \cdot T_E(s)$.
  The Ramp-Input will be $R=2\text{V/sec}$ (i.e.: $R=2$ in the equation for $V_{error}(s)$).

Step 3 – Derive the Steady-State Error Function:

- Apply the Laplace Final Value Theorem (FVT) to $V_{error}(s)$ to derive the formula for steady-state error, $V_{error-ss}(s)$.

Step 4 – Compute $K_a$ for Specified Steady-State Error:

- The formula for steady-state error from Step 3 should be a function of only a single variable, $K_a$, since all other parameters are known.
  Compute the value of $K_a$ to produce a steady-state error $V_{error-ss}(s) = 0.2\text{V}$ to the Ramp-Input.

Step 5 – Determine the Input Signal Frequency:

- The Ramp-Input will be a 10 VPP Triangle-wave from the signal generator.
  Compute the period and associated frequency necessary to create a ramp of $R=2\text{V/sec}$. Note that the ramp signal is half of the total period of the Triangle-wave.

<table>
<thead>
<tr>
<th>T</th>
<th>seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>f</td>
<td>Hz</td>
</tr>
</tbody>
</table>

**Marginal Stability**

Step 1 – Derive the Closed-Loop Transfer Function:

- Derive the closed-loop transfer function, $T(s) = V_p(s)/V_{in}(s)$, for the third-order system shown in Figure 15. The transfer function should be derived with the symbols in the block diagram ($\tau_1$, $K_a$, $K_m$, $\tau_m$, $K_P$), and then the values measured in Lab Exercise #1 should be substituted in. Note that the gain, $K_a$, is still a variable.

The theoretical value of $K_a$ to achieve marginal system stability and the resulting system oscillation frequency will be computed by two different methods.

**Method 1 – Routh-Hurwitz Coefficient Test**

Step 1 – Perform the Routh-Hurwitz coefficient test on the third-order denominator polynomial from the closed-loop transfer function, $T(s)$

Step 2 – Find $K_a$ for Marginal Stability:

- Find the value of $K_a$ that makes the first element in the “$s^1$ row” equal zero.
  This is the predicted value of $K_a$ for marginal stability.

| $K_a$ |         |
Step 3 – Find the Theoretical Oscillation Frequency, $\omega$:

- Substitute the value of $K_a$ from Step 2 into the “$s^2$ row”.

With the “$s^1$ row” equal to zero from Step 2, the “$s^2$ row” is called the Auxiliary Equation. If the coefficients in the “$s^2$ row” are $A$ and $B$, the Auxiliary Equation will have the form $As^2 + B$.

- Manipulate the Auxiliary Equation to be in the form $s^2 + \omega^2$, and solve for the theoretical oscillation frequency, $\omega$ in units of rad/sec.

<table>
<thead>
<tr>
<th>$\omega$ rad/sec</th>
<th>$f$ Hz</th>
</tr>
</thead>
</table>

**Method 2 – Mathematical Substitution**

Step 1 – Substitute $s = j\omega$ into the third-order denominator polynomial from the closed-loop transfer function; $T(s)$ becomes $T(j\omega)$

Step 2 – Solve for $K_a$ and $\omega$:

- Separate the Real and Imaginary parts of the complex function, $T(j\omega)$, from Step 1. This will yield two equations, $\text{Re}\{T(j\omega)\}$ and $\text{Im}\{T(j\omega)\}$, with two unknowns, $K_a$ and $\omega$.

  The poles of the system are where the denominator of the transfer function is equal to zero.

- Solve for the values of $K_a$ and $\omega$ to make both equations equal zero:
  
  $\text{Re}\{T(j\omega)\} = 0$, $\text{Im}\{T(j\omega)\} = 0$

<table>
<thead>
<tr>
<th>$K_a$</th>
<th>$\omega$ rad/sec</th>
<th>$f$ Hz</th>
</tr>
</thead>
</table>

**Procedure:**

**Preliminary Preparation**

Step 1 – Perform the **Balance Pre-Amp Output Procedure** (refer to Procedure section, page 31)

Step 2 – Perform the **Zero Op-Amp Output Procedure** (refer to Procedure section, page 43)

**Steady-State Error**

Step 1 – Set the gain, $K_a$, to the value computed in the pre-lab for Steady State Error:

- Use the **Set Gain using an Attenuator Procedure** (refer to Procedure section, page 44)
Step 2 – Connect the system components as shown in Figure 10 (Lab Exercise #2, page 36):
  - The Op-Amp feedback network is set to the 100kΩ resistor.

Step 3 – Function Generator setup:
  - Set the function generator for Triangle-wave output.
  - Set the frequency to the value computed in the pre-lab for Steady State Error.
  - Set the amplitude to 10 Vpp.
  - Use the oscilloscope to verify that the slope of the Triangle-wave is 2V/sec. Adjust the amplitude and/or frequency if necessary to generate a 2V/sec slope. Get reasonably close and record the actual slope, R, of the ramp input.
  - **IF THE WAVEFORM GENERATOR WILL NOT ALLOW A LOW ENOUGH FREQUENCY TO SATISFY THE PRE-LAB SPECIFICATION:**
    - Set the waveform generator to the lowest possible frequency.
    - **Re-compute** the necessary peak-to-peak amplitude for this period to achieve a 2V/sec ramp. (Recall that the ramp signal is half of the total period of the Triangle-wave.) Note this deviation in your lab report.
  - **Use the actual value of the ramp when comparing the expected vs. measured values of steady-state-error.**
  - Connect the function generator output to the Op-Amp input, as shown in Figure 10.

Step 4 – Oscilloscope setup:
  - Connect the oscilloscope CH1 input to the Input Voltage, V_in. Adjust the vertical sensitivity (Volt/division) of CH1 so that the entire amplitude of the triangle-wave input fits on the screen.
  - Connect the oscilloscope CH2 input to V_position, the output of the Position Indicator (refer to Figure 10). Adjust the vertical sensitivity of CH2 so that the entire amplitude of the response fits on the screen. **Invert** CH2.
  - Adjust horizontal sweep (seconds/division) until a rising slope input (from the low point on the triangle-wave to the peak) is displayed on the screen.
  - With the two signals aligned vertically, the vertical “gap” between the two waveforms is the steady-state error. This “gap” can be measured with the oscilloscope cursors.
  - The “Math” function of the oscilloscope can also be used to plot the difference (i.e.: error) between the two waveforms. The “difference” waveform will resemble an under-damped square-wave step response. Measure the “delta” between the “top” and “bottom” steady-state values of the “difference” waveform. Note that the “delta” between the “top” and “bottom” steady-state values of the “difference” waveform is actually 2*V_error since the “bottom” edge is the error from the negative going ramp (falling slope of the triangle-wave), and the “top” edge is the error from the positive ramp (rising slope of the triangle-wave).
Step 5 – Alternate method to measure $V_{\text{error}}$:
- Connect the oscilloscope CH1 input to the Error Voltage, $V_{\text{error}}$. This signal is the output (plug 6) of the Op-Amp. The Error signal will resemble an under-damped square-wave step response. Adjust the vertical sensitivity (Volt/division) of CH1 so that the entire amplitude of the signal fits on the screen.
- The overall amplitude of this signal is $2V_{\text{error}}$, as explained is Step 4.

<table>
<thead>
<tr>
<th>Measured:</th>
<th>Analyze:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{\text{error}}$</td>
<td>$%$Error</td>
</tr>
</tbody>
</table>

- Use the Hard Copy feature on the oscilloscope to obtain a print-out of the $V_{\text{error}}$ trace.

**Marginal Stability**

Step 1 – Double-Check the Pre-Amp and Op-Amp Balanced/Zeroed Output:
- Perform the **Balance Pre-Amp Output Procedure** (refer to Procedure section, page 31)
- Perform the **Zero Op-Amp Output Procedure** (refer to Procedure section, page 43)

Step 2 – Connect the system components as shown in Figure 16:
- Note that this setup is **EXACTLY THE SAME** as the Steady-State Error exercise **EXCEPT** that the Op-Amp feedback network is set to the RC network ($\tau = 0.1$ sec).
- **Disconnect the Function Generator input.** When the system is unstable, stray noise signals in the system are sufficient to cause the system output to oscillate.
- Connect one oscilloscope channel to $V_{\text{Position}}$, the output of the Position Indicator.
- Connect the other oscilloscope channel to $V_{\text{Motor}}$, the input to the Servo-Amp.

Step 3 – Set the system into Marginal Stability:
- Set $K_a$ to the Maximum value (dial setting 10, gain value 1). The motor position will oscillate with increasing amplitude.
- When the oscillation appears to be about $50^\circ$ peak-to-peak (use $K_p$ to convert from the angular specification to voltage), reduce $K_a$ from the maximum value and observe the $V_{\text{Position}}$ oscillation amplitude.
- Also observe the $V_{\text{Motor}}$ amplitude to find whether the input is within the $\pm V_{\text{MAX}}$ range (for linear – unsaturated – motor operation). [There may also be an audible difference between when the motor is operating in saturated vs. unsaturated modes.]
- When the system is operating at Marginal Stability, the $V_{\text{Position}}$ signal will look like a smooth sine-wave. On the threshold of Marginal Stability, the $V_{\text{Position}}$ amplitude will be constant. At this point, slight changes to $K_a$ will result in either increasing or decreasing $V_{\text{Position}}$ amplitude as the system poles transition between stability and instability.
- Use the Hard Copy feature on the oscilloscope to obtain a print-out of the trace.
• Measure and record the oscillation frequency and compare this to the theoretical value.

• Without disturbing the attenuator setting, disconnect the attenuator from the system and measure the gain, $K_a$, using a method similar to setting the gain to a specific value (refer to Set Gain using an Attenuator Procedure, page 44). Record the measured value of $K_a$ and compare this value to the theoretical value for Marginal Stability.

<table>
<thead>
<tr>
<th>Measured:</th>
<th>Analyze:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'$ Hz</td>
<td>%Error</td>
</tr>
<tr>
<td>$K_a$</td>
<td>%</td>
</tr>
</tbody>
</table>

**Summary:**
Lab Exercise Checklist – Be sure that the following have been obtained in the Lab to complete the lab exercise:

- Collected data supporting the measurements of $V_{error}$ to a Ramp-input for steady-state error exercise, and oscillation frequency and associated $K_a$ for marginal stability exercise
- Annotated hardcopies of step response traces for both $V_{error}$ and un-damped oscillation

Lab Report Checklist – The Lab Report should contain the following supporting documentation:

- All collected data and hardcopies of scope traces
- Derivation of closed-loop transfer functions
- Pre-lab computations
- Error analysis of the measured vs. expected values:
  - Steady-state error, $e_{ss}(V_{error})$ – considering actual ramp-input slope
  - $f'(\omega)$ and $K_a$ for marginal stability

**Review Questions – Answers must be included in the lab report:**
•
Lab Exercise 4 – Design of Position Control System with Velocity Feedback

**Purpose:**
The purpose of this lab exercise is to design and evaluate a position control system, and to improve the ability to control the servo motor system step-response over the position control system analyzed in Lab Exercise #2. In this exercise, an inner velocity feedback loop will be introduced to the system around the motor and tachometer with an additional gain parameter, $K_b$.

![Figure 17 - Position Control System with Velocity Feedback Loop - Block Diagram](image17)

![Figure 18 - Position Control System with Velocity Feedback Loop - Wiring Diagram](image18)
Objectives:
After completing this exercise, you will be able to:
- Design a system’s step-response using two adjustable gain parameters

Reference Reading:
- References from Previous Lab Exercises:
  - Block-Diagram Simplification
  - Second-Order Dynamic Response Design Procedure & Equations
  - Steady-State Error

Velocity Feedback:
- Chapter 5, pp 235-237

Equipment List:
The following pieces of lab equipment will be required to complete this exercise:
- Pre-Amp, Servo-Amp, Motor, and Tachometer “core”
- Position Indicator
- Dual Attenuator
- Operational Amplifier
- Signal Generator
- Oscilloscope
- Printer
- Three coaxial cables with BNC to clip (alligator or microprobe)
- Various interconnect wires

Background Information:
In Lab Exercise #2, a closed-loop position control system was designed to meet a single step response performance requirement: Overshoot, $M_p$, of 25%. The controller was a single variable gain (multiplier) of the error (difference between system input and output). With a single adjustable parameter (one “degree of freedom”), only one performance requirement could be specified.

In this exercise, an inner “velocity loop” will be added; the velocity signal from the tachometer, $V_{tach}$, will be fed-back to an input of the Pre-Amp after first passing through a new adjustable gain, $K_b$.

The second adjustable parameter will result in a system with two degrees of freedom, and will allow the design requirements to specify two step response performance criteria.

Pre-Lab:
Step 1 – Derive the Closed-Loop Transfer Function:
- Derive the closed-loop transfer function, $T(s) = \frac{V_p(s)}{V_{in}(s)}$, for the system shown in Figure 17. The transfer function should be derived with the symbols in the block diagram ($K_a$, $K_b$, $K_m$, $\tau_m$, $K_t$, $K_p$), and then the values measured in Lab Exercise #1 should be substituted in. Note that the two gains, $K_a$ and $K_b$, are still variables.
Additionally, it may be convenient to algebraically manipulate the transfer function so that the coefficient of the $s^2$ term in the denominator is 1.

Step 2 – Specify a Transient Response and Compute $K_a$ and $K_b$ for the Specified Behavior:

- **Specify** (i.e.: choose) an **overshoot, $M_p < 20\%$**, and **Settling Time, $t_{s,2\%} < 4/3$ seconds** to a step input.
- **Determine the value of $K_a$ and $K_b$ that will yield the specified transient response ($M_p$, and $t_{s,2\%}$) to a step input for the transfer function derived in Step 1 of the pre-lab, and use the second-order system design equations as needed.**
- **Predict** the peak time, $t_p$, for this design.
- **Confirm that for a Unit-Ramp input ($1/s^2$ – slope is 1 Volt/sec) the Steady-State Error, $e_{ss} < 4\degree$ (use $K_p$ to convert from the angular specification to voltage).**

**Design Objectives:**

<table>
<thead>
<tr>
<th>$M_p$</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_s$</td>
<td>seconds</td>
</tr>
</tbody>
</table>

**Controller Values:**

<table>
<thead>
<tr>
<th>$K_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_b$</td>
</tr>
</tbody>
</table>

**Expected Peak-Time and Ramp-input steady-state error:**

<table>
<thead>
<tr>
<th>$t_p$</th>
<th>seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{ss}$</td>
<td>degree Volt</td>
</tr>
</tbody>
</table>

**Procedure:**

**Step 1 – Preliminary Preparation:**

- Perform the **Balance Pre-Amp Output Procedure** (refer to Procedure section, page 31)
- Perform the **Zero Op-Amp Output Procedure** (refer to Procedure section, page 43)

**Step 2 – Set the gains, $K_a$ and $K_b$, to the values computed in the pre-lab:**

- Use the **Set Gain using an Attenuator Procedure** (refer to Procedure section, page 44)

**Step 3 – Connect the system components as shown in Figure 18:**

- The Op-Amp feedback network is set to the $100\Omega$ resistor.
Step 4 – Function Generator setup:
- Set the function generator for Square-wave output, frequency low enough so that the step response has sufficient settling time before the next edge of the square-wave (use the settling time, $t_s$, as a guide), amplitude for 50° peak-to-peak. Use $K_p$ to convert from the angular specification to voltage. Note that each edge of the square-wave represents half of the period of the overall waveform.
- Connect the function generator output to the Op-Amp input, $V_{in}$, as shown in Figure 18.
- Confirm that the $V_{Motor}$ input to the servo-amp is limited to $\pm V_{MAX}$ Vpp; reduce $V_{in}$ if necessary.

Step 5 – Oscilloscope setup:
- Connect the oscilloscope CH1 input to the Input Voltage, $V_{In}$.
  Adjust the vertical sensitivity (Volt/division) of CH1 so that the entire amplitude of the square-wave input fits on the screen.
- Connect the oscilloscope CH2 input to $V_{Position}$, the output of the Position Indicator (refer to Figure 18).
  Adjust the vertical sensitivity of CH2 so that the entire amplitude of the step response fits on the screen. Invert CH2.
- Adjust horizontal sweep (seconds/division) until a single step response (from the beginning of the step until the final value is reached) is displayed on the screen.

Step 6 – Measure and Record the Step Response characteristics:
- Perform the Measure Characteristics of an Under-Damped Step Response Procedure (refer to Procedure section, page 45).
- Use the Hard Copy feature on the oscilloscope to obtain a print-out of the step response.
- Record the measured values of $t_p$, $t_s$ and $M_p$ below and compare the measured values with the expected values computed in the second part of the pre-lab:

<table>
<thead>
<tr>
<th>Measured:</th>
<th>Analyze:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_p$ seconds</td>
<td>% Error</td>
</tr>
<tr>
<td>$t_s$ seconds</td>
<td>%</td>
</tr>
<tr>
<td>$M_p$</td>
<td>%</td>
</tr>
</tbody>
</table>

- On the hard copy/print-out of step response, indicate $t_p$, $t_s$ and $M_p$.

The Steady-State Analysis to a Ramp-Input is only theoretical – not part of the Lab Exercise.
Summary:
Lab Exercise Checklist – Be sure that the following have been obtained in the Lab to complete the lab exercise:
- Collected data supporting the measurements of $t_p$, $t_s$ and $M_p$
- Annotated hardcopy of step response

Lab Report Checklist – The Lab Report should contain the following supporting documentation:
- All collected data and hardcopies of scope traces
- Derivation of closed-loop transfer function
- Pre-lab computations
- Error analysis of the measured vs. expected values of $t_p$, $t_s$ and $M_p$
- Theoretical analysis of steady-state error to a ramp-input

Review Questions – Answers must be included in the lab report:
- How was the addition of the velocity feedback able to allow two step response performance specifications to be met?
Lab Exercise 5 – System Stabilization Design by Velocity Feedback

Purpose:
The purpose of this lab exercise is to stabilize an unstable system (third-order system from Lab Exercise #3 – intentionally made unstable) using the inner velocity feedback loop that was introduced in Lab Exercise #4.

Figure 19 - Third-Order System with Velocity Feedback Loop - Block Diagram

Figure 20 – Third-Order System with Velocity Feedback Loop - Wiring Diagram
**Objectives:**
After completing this exercise, you will be able to:
- Stabilize an unstable system’s step-response using velocity feedback

**Reference Reading:**
- References from Previous Lab Exercises:
  - Block-Diagram Simplification
  - Second-Order Dynamic Response Design Procedure & Equations
  - Stability

**Equipment List:**
The following pieces of lab equipment will be required to complete this exercise:
- Pre-Amp, Servo-Amp, Motor, and Tachometer “core”
- Position Indicator
- Dual Attenuator
- Operational Amplifier
- Signal Generator
- Oscilloscope
- Printer
- Three coaxial cables with BNC to clip (alligator or microprobe)
- Various interconnect wires

**Background Information:**
In Lab Exercise #3, third pole was introduced to the system. The additional pole adds phase lag (delay). With the additional lag, increasing the system gain above some threshold value caused the system to become unstable.

In this exercise, the inner “velocity loop” introduced in Lab Exercise #4 will be added to the third-order position control system from Lab Exercise #3. The gain $K_a$ will be set to a value to intentionally cause system instability. Then, the velocity feedback gain, $K_b$, will be designed so as to return the system to marginal stability, and then to an over-damped condition.

**Pre-Lab:**

**Preliminary**
Step 1 – Derive the Closed-Loop Transfer Function:
- Derive the closed-loop transfer function, $T(s) = V_{\text{Position}}(s)/V_{\text{In}}(s)$, for the system shown in Figure 19. The transfer function should be derived with the symbols in the block diagram ($K_a$, $\tau_1$, $K_b$, $K_m$, $\tau_m$, $K_t$, $K_p$), and then the values measured in Lab Exercise #1 should be substituted in. Note that the two gains, $K_a$ and $K_b$, are still variables.
Marginal Stability
Step 1 – Induce System Instability:
• Begin with the value of $K_a$ from Lab Exercise #3 that was found to bring the system into marginal stability.
• Increase that value of $K_a$ by 10% to assure that the system is unstable.

Step 2 – Find $K_b$ for Marginal Stability and the Theoretical Oscillation Frequency, $\omega$:
• Using either method from Lab Exercise #3, compute the value of $K_b$ that restores the system to marginal stability, and also find the theoretical oscillation frequency, $\omega$ in units of rad/sec.

Response for $K_b = 1$
Step 1 – Predict Transient Response for $K_b = 1$:
• Let $K_b = 1$ in the closed-loop transfer function, and keep the same value of $K_a$ (unstable) from the previous part of this exercise.
• Predict the step-response performance (peak time, $t_p$, settling time, $t_s$, and percent overshoot, $M_p$) and the steady-state error, $e_{ss}$, to a unit-ramp input ($1/s^2$ – slope is 1 Volt/sec).

Procedure:
Step 1 – Preliminary Preparation:
• Perform the Balance Pre-Amp Output Procedure (refer to Procedure section, page 31)
• Perform the Zero Op-Amp Output Procedure (refer to Procedure section, page 43)

Step 2 – Set the gains, $K_a$ and $K_b$, to the values computed in the pre-lab:
• Use the Set Gain using an Attenuator Procedure (refer to Procedure section, page 44)
Step 3 – Connect the system components as shown in Figure 20:

- The Op-Amp feedback network is set to the RC network (τ = 0.1 sec).
- Note that the signal generator input is not needed to maintain the marginally stable (i.e.: un-damped) oscillation, and should be disconnected from the V_{in} input for this first part of the lab exercise.

Step 4 – Set the system into Marginal Stability:

- Use the same technique described in Lab Exercise #3 to set the system into a marginally stable state.
- Use the Hard Copy feature on the oscilloscope to obtain a print-out of the undamped system oscillation.
- Measure and record the value of K_b that resulted in marginal stability.
- Measure and record the oscillation frequency of the marginally stable system.

<table>
<thead>
<tr>
<th>Measured:</th>
<th>Analyze:</th>
</tr>
</thead>
<tbody>
<tr>
<td>K_b</td>
<td>%Error</td>
</tr>
<tr>
<td>f</td>
<td>Hz</td>
</tr>
</tbody>
</table>

Step 5 – Set K_b to 1:

- Replace the attenuator for K_b with a direct connection from the tachometer output to the pre-amp input. This sets the value of K_b to 1.

Step 6 – Function Generator setup:

- Set the function generator for Square-wave output, frequency low enough so that the step response has sufficient settling time before the next edge of the square-wave (use the predicted settling time, t_s, as a guide), amplitude for 50° peak-to-peak. Use K_p to convert from the angular specification to voltage.
- Note that each edge of the square-wave represents half of the period of the overall waveform.
- Connect the function generator output to the Op-Amp input, V_{in}, as shown in Figure 20.
- Confirm that the V_{Motor} input to the servo-amp is limited to ±V_{MAX} Vpp; reduce V_{in} if necessary.

Step 7 – Measure and Record the Step Response characteristics:

- Measure and record the step response characteristics and compare these to the predictions from the pre-lab.
- Perform the Measure Characteristics of an Under-Damped Step Response Procedure (refer to Procedure section, page 45).
- Use the Hard Copy feature on the oscilloscope to obtain a print-out of the step response.
<table>
<thead>
<tr>
<th>Measured:</th>
<th>Analyze:</th>
</tr>
</thead>
<tbody>
<tr>
<td>t_p</td>
<td>seconds</td>
</tr>
<tr>
<td>t_s</td>
<td>seconds</td>
</tr>
<tr>
<td>M_p</td>
<td>%</td>
</tr>
</tbody>
</table>

- On the hard copy/print-out of step response, indicate $t_p$, $t_s$ and $M_p$.

The Steady-State Analysis to a Ramp-Input is only theoretical – not part of the Lab Exercise.

Summary:
Lab Exercise Checklist – Be sure that the following have been obtained in the Lab to complete the lab exercise:
- Collected data supporting the measurements of oscillation frequency and associated $K_b$ for marginal stability, and step response when $K_b = 1$
- Annotated hardcopies of un-damped system oscillation and step response for $K_b = 1$

Lab Report Checklist – The Lab Report should contain the following supporting documentation:
- All collected data and hardcopies of scope traces
- Derivation of closed-loop transfer function
- Pre-lab computations
- Error analysis of the measured vs. expected values of $K_b$, un-damped oscillation frequency, and step response for $K_b = 1$
- Theoretical analysis of steady-state error to a ramp-input

Review Questions – Answers must be included in the lab report:
- 

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Prof. B. D. Sweet  
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April 24, 2005
Lab Exercise 6 – Phase Lead Compensator Design

**Purpose:**
The purpose of this lab exercise is to design a phase lead compensator to meet the specified time-domain performance requirements.

![Position Controller with Lead Compensator - Block Diagram](image1)

**Figure 21 - Position Controller with Lead Compensator - Block Diagram**

![Position Controller with Lead Compensator - Wiring Diagram](image2)

**Figure 22 – Position Controller with Lead Compensator - Wiring Diagram**
Objectives:
After completing this exercise, you will be able to:

- Design and implement a Phase-Lead compensator to meet specified system time response specifications

Reference Reading:

- References from Previous Lab Exercises:
  - Second-Order Dynamic Response Design Procedure & Equations
  - Steady-State Error

  Cascade Compensation Networks:
  - Chapter 10, pp 557-560
  Phase-Lead Design:
  - Chapter 10, pp 567-573

  Selecting the Parameter Value, and Dynamic Compensation:
  - Chapter 5, pp 307-316
  Analog and Digital Implementation:
  - Chapter 5, pp 320-322

  Lead Compensation, Lead-Lag Compensation:
  - Chapter 9, pp 524-530, 537-542, 564-566 (case study)
  Physical Realization of Compensation:
  - Chapter 9, pp 558-563

  Lead Compensation:
  - Chapter 7, pp 421-429

Equipment List:
The following pieces of lab equipment will be required to complete this exercise:

- Pre-Amp, Servo-Amp, Motor, and Tachometer “core”
- Position Indicator
- Dual Attenuator
- Operational Amplifier
- Signal Generator
- Oscilloscope
- Printer
- Three coaxial cables with BNC to clip (alligator or microprobe)
- Various interconnect wires
- Discrete components to build the Lead compensator network

Background Information:
In Lab Exercises #2, 3, 4, and 5, variable gain parameters, $K_a$ and $K_b$, were adjusted such that the closed-loop system met the time-domain performance specifications. In all of
these systems, the root-loci of the closed-loop systems passed through regions of the s-plane such that the specifications could be met by merely adjusting gain values.

In this lab exercise, the time-domain performance specifications require closed-loop pole locations that do not lie on the system’s root locus, and therefore cannot be met by adjusting gain values alone.

To meet the performance specifications, it will be necessary to design a compensator with a pole and a zero along with a gain parameter. The additional pole and zero from the compensator will “bend” the root locus, or “pull” the closed-loop system poles, into a region of the s-plane that can meet the desired performance specifications. A Phase-Lead compensator has its zero closer to the origin of the s-plane than its pole, as shown in Figure 23. It has the general transfer function:

\[ G_c(s) = K_c \frac{(\tau_z s + 1)}{(\tau_p s + 1)} \]

A phase-lead compensator may be constructed from discrete resistors and capacitors as follows (references [2] and [4]):

![Figure 23 – Phase-Lead Compensator Pole-Zero Plot](image)

![Figure 24 - R-C Phase-Lead Network](image)
The R-C circuit has the following transfer function:

\[ G_c(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R_2}{R_2 + \left( R_1 \parallel \frac{1}{sC} \right)} = \left( \frac{R_2}{R_1 + R_2} \right) \left( \frac{R_1 R_2}{R_1 + R_2} \right) C \cdot s + 1 \]

\[ \equiv \left( \frac{1}{\alpha} \right) \left( \frac{\alpha \tau \cdot s + 1}{(\tau \cdot s + 1)} \right) \]

Where:

\[ \tau = \left( \frac{R_1 R_2}{R_1 + R_2} \right) C \]

\[ \alpha = \left( \frac{R_1 + R_2}{R_2} \right) \]

OR

\[ G_c(s) \equiv \beta \cdot \left( \frac{\tau \cdot s + 1}{(\beta \tau \cdot s + 1)} \right) \]

Where:

\[ \beta = \left( \frac{R_2}{R_1 + R_2} \right) \]

In the first formulation, note that \( \alpha \) is greater than 1; the zero at \( \frac{1}{\alpha \tau} \) is closer to the origin than the pole at \( \frac{1}{\tau} \).

In the second formulation, note that \( \beta \) is less than 1; the pole at \( \frac{1}{\beta \tau} \) is further from the origin than the zero at \( \frac{1}{\tau} \).

Note also that the D.C. gain of the system (transfer function gain with \( s = 0 \)) is:

\[ \frac{1}{\alpha} = \beta = \frac{R_2}{R_1 + R_2} \]

The values \( \alpha \) and \( \beta \) can be thought of as separation ratios, or separation distances, between the compensator pole and zero. \( \alpha \) typically has a value between 3 and 20. One “rule of thumb” suggests that the pole be at least ten-times further than the zero from the s-plane origin \((p \geq 10z)\). Note, however that large values of \( \alpha \) also result in increased attenuation from the voltage divider.
A phase-lead compensator may also be constructed from an op-amp circuit as follows (reference [2]):

![Op-Amp Phase-Lead Circuit](image)

*Figure 25 – Op-Amp Phase-Lead Circuit*

The op-amp circuit has the following transfer function:

\[
G_c(s) = \frac{V_{out}(s)}{V_{in}(s)} = -\frac{R_F}{R_2 + \left( \frac{1}{R_1 s C} \right)} \equiv \left( \frac{R_F}{R_1 + R_2} \right) \cdot \frac{R_1 R_2 (sC + 1)}{R_1 R_2 (s + \frac{1}{R_1 C})}
\]

\[
\equiv -K_c \cdot \frac{\alpha \tau \cdot s + 1}{\tau \cdot s + 1}
\]

Where:

\[
\alpha = \frac{R_1 + R_2}{R_2}, \quad \tau = \frac{R_1 R_2}{R_2 (s + \frac{1}{R_1 C})}, \quad K_c = \left( \frac{R_F}{R_1 + R_2} \right)
\]

OR

\[
G_c(s) \equiv -K_c \cdot \frac{(\beta \tau \cdot s + 1)}{(\tau \cdot s + 1)}
\]

Where:

\[
\beta = \frac{R_2}{R_1 + R_2}, \quad \tau = (R_1 C), \quad K_c = \left( \frac{R_F}{R_1 + R_2} \right)
\]

A second inverting op-amp can be added to reverse the sign inversion.
Pre-Lab:
Step 1 – Derive the Closed-Loop Transfer Function:
- Derive the closed-loop transfer function, $T(s) = \frac{V_{\text{Position}}(s)}{V_{\text{In}}(s)}$, for the system shown in Figure 21. The transfer function should be derived with the symbols in the block diagram ($K_c$, $\tau_z$, $\tau_p$, $K_m$, $\tau_m$, $K_p$), and then the values measured in Lab Exercise #1 should be substituted in. Note that the compensator parameters, $K_c$, $\tau_z$, and $\tau_p$, are still variables.

Step 2 – Set Design Objectives:
- Specify step response design objectives to meet the following requirements:
  - Percent Overshoot, $M_p < 25\%$
  - Step response Settling Time (2%) < 0.8 sec

- For Unit-Ramp input ($1/s^2$ – slope is 1 Volt/sec) the Steady-State Error, $e_{ss} < 1^\circ$ (use $K_p$ to convert from the angular specification to voltage).

Design Objectives:

<table>
<thead>
<tr>
<th>$M_p$</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_s$</td>
<td>seconds</td>
</tr>
<tr>
<td>$e_{ss}$</td>
<td>Volt</td>
</tr>
</tbody>
</table>

Step 3 – Specify Desired Closed-Loop Dominant Pole Location to meet Design Objectives:
- Use the second-order system design equations to determine the location of the desired dominant closed-loop system poles.

Desired Dominant Closed-Loop Pole Pair:

<table>
<thead>
<tr>
<th>$s$</th>
</tr>
</thead>
</table>

Step 4 – Design the Compensator Pole, Zero, and Gain values to meet Design Objectives:
- Refer to the references reading material and “Lab Exercise 6 – Appendix – Design Pole, Zero, and Gain Values for Phase-Lead Compensator” for guidance.

Your design assumptions, calculations, etc. must be included in your lab report.

Discussion:
The overall system will be third-order. The design equations will be applied to the dominant complex conjugate pole pair. The third pole, $p_3$, must be real. The resulting closed-loop transfer function denominator will therefore be:

$$(s + p_3)(s^2 + 2\zeta \omega_n \cdot s + \omega_n^2) = s^3 + (p_3 + 2\zeta \omega_n) s^2 + (2\zeta \omega_n p_3 + \omega_n^2) s + (p_3 \cdot \omega_n^2)$$
Use the second-order system design equations to determine the necessary $\zeta$ and $\omega_n$ values, and substitute these into the expanded third-order equation above. (The location of the third pole, $p_3$, is still a variable.)

### Compensator Pole, Zero, and Gain:

| $p_c$ |  |
| $p_z$ |  |
| $K_c$ |  |

- Your completed design should include an evaluation to assure that the theoretical closed loop step response will meet the Design Objectives. Simulation tools such as MatLab and MathCad may be used here.
- **Predict** the peak time, $t_p$, for this design.

### Expected Peak-Time and Ramp-input steady-state error:

| $t_p$ | seconds |
| $e_{ss}$ | degree Volt |

### Procedure:

**Step 1 – Preliminary Preparation:**

- Perform the **Balance Pre-Amp Output Procedure** (refer to Procedure section, page 31)
- Perform the **Zero Op-Amp Output Procedure** (refer to Procedure section, page 43)

**Step 2 – Connect the system components as shown in Figure 22:**

- The Op-Amp feedback network is set to the 100KΩ resistor.
- Build and insert the Compensator circuit with the designed values and place the Compensator circuit in the appropriate location.
Step 3 – Function Generator setup:

- Set the function generator for Square-wave output, frequency low enough so that the step response has sufficient settling time before the next edge of the square-wave (use the settling time, $t_s$, as a guide), amplitude for 50° peak-to-peak. Use $K_p$ to convert from the angular specification to voltage. Note that each edge of the square-wave represents half of the period of the overall waveform.
- Connect the function generator output to the Op-Amp input, $V_{in}$, as shown in Figure 22.
- Confirm that the $V_{Motor}$ input to the servo-amp is limited to $\pm V_{MAX} \text{ Vpp}$; reduce $V_{in}$ if necessary.

Step 4 – Oscilloscope setup:

- Connect the oscilloscope CH1 input to the Input Voltage, $V_{in}$.
  Adjust the vertical sensitivity (Volt/division) of CH1 so that the entire amplitude of the square-wave input fits on the screen.
- Connect the oscilloscope CH2 input to $V_{Position}$, the output of the Position Indicator (refer to Figure 22).
  Adjust the vertical sensitivity of CH2 so that the entire amplitude of the step response fits on the screen.
  **Invert CH2.**
- Adjust horizontal sweep (seconds/division) until a single step response (from the beginning of the step until the final value is reached) is displayed on the screen.

Step 5 – Measure and Record the Step Response characteristics:

- Perform the **Measure Characteristics of an Under-Damped Step Response Procedure** (refer to Procedure section, page 45).
- Use the Hard Copy feature on the oscilloscope to obtain a print-out of the step response.
- Record the measured values of $t_p$, $t_s$ and $M_p$ below and compare the measured values with the design values in the pre-lab:

<table>
<thead>
<tr>
<th>Measured:</th>
<th>Analyze:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_p$</td>
<td>seconds</td>
</tr>
<tr>
<td>$t_s$</td>
<td>seconds</td>
</tr>
<tr>
<td>$M_p$</td>
<td>%</td>
</tr>
</tbody>
</table>

- On the hard copy/print-out of step response, indicate $t_p$, $t_s$ and $M_p$.

**The Steady-State Analysis to a Ramp-Input is only theoretical – not part of the Lab Exercise.**
Summary:
Lab Exercise Checklist – Be sure that the following have been obtained in the Lab to complete the lab exercise:
- Collected data supporting the measurements of $t_p$, $t_s$ and $M_p$
- Annotated hardcopy of step response

Lab Report Checklist – The Lab Report should contain the following supporting documentation:
- All collected data and hardcopies of scope traces
- Derivation of closed-loop transfer function
- Pre-lab computations INCLUDING compensator design (pole, zero, gain, and circuit components)
- Error analysis of the measured vs. expected values of $t_p$, $t_s$ and $M_p$
- Theoretical analysis of steady-state error to a ramp-input

Review Questions – Answers must be included in the lab report:
- In what ways is Phase-Lead compensation superior to compensation using gain alone?
Lab Exercise 6 – Appendix – Design Pole, Zero, and Gain Values for Phase-Lead Compensator

**Purpose:**
This section describes some techniques and background information that may be helpful in solving for the necessary Pole, Zero, and Gain values for a Phase-Lead Compensator to yield closed-loop system poles at specified locations.

![Diagram of s-plane with poles and zeros](image)

**Figure 26 - System and Compensator Poles**

There are various methods to determine the compensator pole and zero locations; some of these are discussed in the reference reading material for the lab exercise.

**All methods have the following in common:**

**Angle Criteria**
The angle criteria of the closed-loop root locus requires that the sum of the angles of the open-loop zeros (with respect to the desired closed-loop pole location) minus the sum of the open-loop poles (with respect to the desired closed-loop pole location) must equal an integer multiple of 180° (refer to Figure 26):

$$\angle G(s) = \angle \frac{\prod_{i}(s + z_i)}{\prod_{j}(s + p_j)} = \sum_{i} \angle(s + z_i) - \sum_{j} \angle(s + p_j) \equiv 180^\circ \pm k \cdot 360^\circ$$

where ‘s’ is the desired closed-loop pole location.
For the system with poles and zeros shown in Figure 26, this would be:

\[ \theta_{zc} - (\theta_{pc} + \theta_{p2} + \theta_{p1}) = -180^\circ \]

**Magnitude Criteria**

The magnitude criteria of the closed-loop root locus requires that the product of the pole magnitudes (distances from the open-loop poles to the desired closed-loop pole location) divided by the product of the zero magnitudes (distances from the open-loop zeros to the desired closed-loop pole location) is equal to the open-loop gain, K.

For a closed-loop system with the transfer function denominator:

\[ 1 + K \cdot G(s) = \prod_{i} (s + z_i) \]

The Magnitude Criteria requires that:

\[ \prod_{j} |s + p_j| = \prod_{i} |s + z_i| \]

where ‘s’ is the desired closed-loop pole location.

Note that the magnitude criteria will give the OVERALL system gain, K, that is necessary to place the closed-loop pole at the desired location. Therefore, to find the compensator gain, K_c, the overall system gain must be divided by the open-loop D.C. gain of the original uncompensated system.

**Common methods for finding compensator poles and zeros:**

**Arbitrary placement of Compensator Zero**

Step 1 – Place the Compensator Zero

- The placement of the compensator zero is arbitrary. Placing the zero below or to the left of the desired closed-loop pole location may help to assure that the desired closed-loop pole (and it’s complex conjugate) are the dominant system poles.
- Some of the reference reading material discusses various strategies for placing the zero.

Step 2 – Solve for the Compensator Pole

- Use the root-locus angle criteria to determine the required angle of the compensator pole, \( \theta_{pc} \) (refer to Figure 26).
- Use trigonometric identities to solve for the placement of the pole to meet the angle criteria.

Step 3 – Solve for the Compensator Gain

- Use the root-locus magnitude criteria with the system and compensator zeros and poles to solve for the compensator gain, K_c.
Simultaneous Solution of Compensator Pole and Zero

There are various simultaneous solutions that can be considered. One is to use the root locus angle criteria to determine the overall phase that the compensator must add:

- Use the root-locus angle criteria to determine the overall angle required for the compensator, $\theta_c$ (refer to Figure 26):

$$\theta_c - ( \theta_{p2} + \theta_{p1} ) = -180^\circ$$, where $\theta_c = ( \theta_{zc} - \theta_{pc} )$

- The location of the compensator pole and zero are found using geometric and trigonometric identities to satisfy the angle criteria.

The following techniques may be useful:

Technique: Measuring Pole and Zero Angles

The angles from open-loop poles and zeros to the desired closed-loop pole location can be “measured” using basic trigonometric identities.

The general relationship for the angles of open-loop poles or zeros and the desired closed-loop pole location is:

$$\tan(\theta_p) = \frac{\text{Im}\{s\} - \text{Im}\{p\}}{\text{Re}\{s\} - \text{Re}\{p\}}$$
The angles of poles and zeros to the left of the desired closed-loop pole location can be “measured” using the identity:

\[
\theta = \tan^{-1}\left(\frac{\text{Im}\{s\} - \text{Im}\{p\}}{\text{Re}\{s\} - \text{Re}\{p\}}\right)
\]

For the pole \(p_2\) in Figure 27, the angle is found by:

\[
\theta_{p_2} = \tan^{-1}\left(\frac{\omega}{p_2 - \sigma}\right)
\]

The angles for poles and zeros to the right of the desired closed-loop pole location can be “measured” using the identity:

\[
\phi = \tan^{-1}\left(\frac{\text{Im}\{s\} - \text{Im}\{p\}}{\text{Re}\{s\} - \text{Re}\{p\}}\right), \quad \text{and} \quad \theta = 180^\circ - \phi
\]

For the pole \(p_1\) in Figure 27, the angle is found by:

\[
\theta_{p_1} = 180^\circ - \phi_{p_1} = 180^\circ - \tan^{-1}\left(\frac{\omega}{\sigma - p_1}\right)
\]

**Technique: Placing the Pole with the Given Angle**

The placement of the open-loop pole with respect to the desired closed-loop pole location given the angle can be found using basic trigonometric identities.

For the pole \(p_2\) in Figure 27, the placement of the pole given the angle is found by:

\[
\tan(\theta_p) = \frac{y}{x} = \frac{\text{Im}\{s\} - \text{Im}\{p\}}{\text{Re}\{s\} - \text{Re}\{p\}}
\]

\[
\Rightarrow \tan(\theta_{p_2}) = \frac{\omega}{p_2 - \sigma}
\]

\[
\Rightarrow p_2 = \frac{\omega}{\tan(\theta_{p_2})} + \sigma
\]

**Technique: Measuring OVERALL System Gain**

The necessary OVERALL system gain necessary to place the closed-loop pole at the desired location can be “measured” using the root locus magnitude criteria and basic trigonometric identities.

From the root locus magnitude criteria:

\[
K = \frac{\prod |s + p_j|}{\prod |s + z_i|}, \quad \text{where} \; \text{‘s’ is the desired closed-loop pole location.}
\]
The magnitude (distance from the open-loop pole or zero to the desired closed-loop pole) can be considered as the hypotenuse of a right triangle:

\[
K = \frac{\prod \sqrt{(\text{Re}\{s\} - \text{Re}\{p_i\})^2 + (\text{Im}\{s\} - \text{Im}\{p_i\})^2}}{\prod \sqrt{(\text{Re}\{s\} - \text{Re}\{z_i\})^2 + (\text{Im}\{s\} - \text{Im}\{z_i\})^2}}
\]

where ‘s’ is the desired closed-loop pole location.

For the system with poles and zeros shown in Figure 26, and the desired close-loop pole location is \( s = -\sigma + j\omega \), this would be:

\[
K = \sqrt{\frac{(\sigma - p_1)^2 + (\omega)^2 \cdot (\sigma - p_2)^2 + (\omega)^2 \cdot (\sigma - p_{c})^2 + (\omega)^2}{(\sigma - z_c)^2 + (\omega)^2}}
\]

**Note that OVERALL system gain, \( K \), must be divided by the open-loop D.C. gain of the original uncompensated system to find the necessary compensator gain, \( K_c \).**
Lab Exercise 7 – Control System Frequency Response Analysis

Purpose:
The purpose of this lab exercise is to analyze the frequency response of the subject systems, and to collect and plot frequency-domain information of the systems.

The fist part of the exercise evaluates the position control system from Lab Exercise #2 (Figure 9 and Figure 10, page 36), and uses the value of $K_a$ computed to achieve a 25% overshoot to a step input. This closed-loop second-order system is then explored by comparing sinusoidal input and output signals and constructing the system Bode plot.

The second part of the exercise, evaluates the marginally stable OPEN-LOOP third-order system (Figure 28 and Figure 29, page 82) (system from Lab Exercise #3 without the feedback), and uses the value of $K_a$ measured in Lab Exercise #3 to achieve a marginally stable system. This open-loop third-order system is then explored by comparing sinusoidal input and output signals and constructing the system Nyquist (polar) plot.

Objectives:
After completing this exercise, you will be able to:

- Perform sinusoidal steady-state analysis of a system
- Sketch a Bode Plot from laboratory data of sinusoidal system input and output signals.
- Sketch a Nyquist Plot from laboratory data of sinusoidal system input and output signals.

Reference Reading:
   Bode Analysis:
   - Chapter 15, pp 364-383
   Nyquist Analysis:
   - Chapter 11, pp 246-260
   Frequency Response and Bode:
   - Chapter 8, pp 406-442
   The Nyquist Criterion:
   - Chapter 9, pp 476-493
   Frequency Response and Bode Plot Techniques:
   - Chapter 6, pp 364-384
   The Nyquist Stability Criterion:
   - Chapter 6, pp 390-409
   Frequency Response Techniques and Bode Plots:
     o Chapter 10, pp 590-618
   Introduction to the Nyquist Criterion:
     o Chapter 10, pp 619-635
   Frequency-Response Analysis and Bode Diagrams:
     o Chapter 8, pp 492-521
   Polar Plots, Nyquist Plots, and Nyquist Stability Criterion:
     o Chapter 8, pp 523-556

Additional Reference:
GraphTablet – “Gives you a never ending supply of simple graph paper from your home printer.” This utility allows you to generate custom semi-log grids for Bode plots and polar grids for Nyquist plots.
http://www.graphtablet.com/
Figure 28 – Open-Loop Third-Order System Block Diagram for Nyquist Analysis

Figure 29 – Open-Loop Third-Order System Wiring Diagram for Nyquist Analysis

**Equipment List:**
The following pieces of lab equipment will be required to complete this exercise:
- Pre-Amp, Servo-Amp, Motor, and Tachometer “core”
- Position Indicator
- Dual Attenuator
- Operational Amplifier
- Signal Generator
- Oscilloscope
- Printer
- Three coaxial cables with BNC to clip (alligator or microprobe)
- Various interconnect wires
**Background Information:**
The previous lab exercises have evaluated systems by means of their “transient” (step) responses. In theory, one could analyze the step response of a system to determine the location of the dominant system poles. This method has two fundamental shortcomings:
- Only dominant system poles are identified. “Faster” poles far into the Left-Half Plane may not be observed.
- The step input has “vertical” discontinuity that would require an infinite amount of energy to realize in a real system. (The square-wave inputs in the previous lab exercises have only been approximations of step inputs.)

Sinusoidal Steady-State system analysis techniques that compare the input and output gain and phase differences have the advantage of being realizable in physical systems. A “black box” system can be identified by applying a series of sinusoidal inputs at different frequencies, and measuring the resulting change in magnitude and phase of the output signal. In fact, there is equipment available that automates this type of system identification.

**Bode** analysis plots both gain magnitude (ratio of output to input amplitude) and phase (difference between output and input phase angles) on two separate, parallel plots with respect to frequency, using a logarithmic scale for angular frequency, \( \omega \). **Nyquist** analysis plots gain magnitude and phase on a single polar plot on the complex s-plane independent of frequency.

**Pre-Lab:**

**Closed-Loop Bode Analysis**

Step 1 – Derive Closed-Loop Transfer Function, Gain, and Angle Formulas:
- Re-state the closed-loop transfer function, \( T(s) = \frac{V_p(s)}{V_{in}(s)} \), for the second order position control system from Lab Exercise #2, as depicted in Figure 9 (page 36). Use the value of gain, \( K_a \), computed in Lab Exercise #2 for a 25% overshoot to a step input.
- Re-write the s-domain closed-loop transfer function as a frequency-domain function by making the substitution \( s = j\omega \); \( T(s) \) becomes \( T(j\omega) \).
- Derive the equations for the magnitude and phase angle of the closed-loop transfer function \( T(j\omega) \) as:

\[
\text{Magnitude, } M = |T(j\omega)| = \frac{|\text{Num}(j\omega)|}{|\text{Den}(j\omega)|} = \sqrt{\left(\text{Re}\{\text{Num}(j\omega)\}\right)^2 + \left(\text{Im}\{\text{Num}(j\omega)\}\right)^2} \sqrt{\left(\text{Re}\{\text{Den}(j\omega)\}\right)^2 + \left(\text{Im}\{\text{Den}(j\omega)\}\right)^2}
\]
**Phase Angle (degrees),** \( \Theta = \angle T(j\omega) = \angle (\text{Num}(j\omega)) - \angle (\text{Den}(j\omega)) \)

\[
\Theta = \tan^{-1}\left(\frac{\text{Im}\{\text{Num}(j\omega)\}}{\text{Re}\{\text{Num}(j\omega)\}}\right) - \tan^{-1}\left(\frac{\text{Im}\{\text{Den}(j\omega)\}}{\text{Re}\{\text{Den}(j\omega)\}}\right) \text{ degrees}
\]

Step 2 – Compute the Natural Frequency, Resonant Frequency, Resonant Peak Magnitude, and Phase Angle of the Closed-Loop Under-damped Second Order System:

- Compute the natural frequency, \( \omega_n \), (the “break frequency”) from the closed-loop transfer function. This may be found from either \( T(s) \) or \( T(j\omega) \).
- Use differential equation techniques to determine the resonant frequency, \( \omega_r \); the value of \( \omega \) that maximizes the magnitude, \( M = |T(j\omega)| \).
  
  Note that this is NOT the observed (damped) oscillation frequency in the time-domain step response, \( \omega_d = \omega_n \sqrt{1 - \zeta^2} \); rather it is \( \omega_r = \omega_n \sqrt{1 - 2\zeta^2} \) - use this formula as a “sanity check” for your derivation.

  Your derivation for \( \omega_r \) must be included in your lab report.

- Substitute \( \omega_r \) into the magnitude and phase angle equations to compute the expected resonant peak magnitude at \( \omega_r \), \( M_r = |T(j\omega_r)| \), and the associated phase angle, \( \theta_r \).

For the system \( \frac{\omega_r^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \), the resonant peak magnitude is \( \frac{1}{2\zeta\sqrt{1 - \zeta^2}} \).

The value of magnitude should then be converted to decibels (dB).

**Expected Natural Frequency, Resonant Frequency, Resonant Peak Magnitude, and Phase Angle:**

| \( \omega_n \) | rad/sec | Hz |
| \( \omega_r \) | rad/sec | Hz |
| \( M_r \) | (raw value) | dB |
| \( \theta_r \) | degrees |

**Open-Loop Nyquist Analysis**

Step 1 – Derive Open-Loop Transfer Function, Gain, and Angle Formulas:

- Re-examine the third-order system from Lab Exercise #3, as depicted in Figure 15 (page 49).
- Derive the OPEN-LOOP transfer function (the “forward path”), for the system depicted in Figure 28 (page 82); \( G(s) = \frac{V_p(s)}{V_{in}(s)} \).
  
  Use the value of gain, \( K_a \), measured in Lab Exercise #3 to achieve marginal system stability.

| \( K_a \) |  |
• Re-write the s-domain open-loop transfer function as a frequency-domain function by making the substitution \( s = j\omega \); \( G(s) \) becomes \( G(j\omega) \).

• Derive the equations for the magnitude and phase angle of the open-loop transfer function \( G(j\omega) \) as:

\[
\text{Magnitude, } M = |G(j\omega)| = \frac{|\text{Num}(j\omega)|}{|\text{Den}(j\omega)|} = \sqrt{\left( \text{Re}\{\text{Num}(j\omega)\} \right)^2 + \left( \text{Im}\{\text{Num}(j\omega)\} \right)^2} \over \sqrt{\left( \text{Re}\{\text{Den}(j\omega)\} \right)^2 + \left( \text{Im}\{\text{Den}(j\omega)\} \right)^2}
\]

\[
\text{Phase Angle (degrees), } \Theta = \angle G(j\omega) = \angle \left( \text{Num}(j\omega) \right) - \angle \left( \text{Den}(j\omega) \right)
\]

\[
= \tan^{-1} \left( \frac{\text{Im}\{\text{Num}(j\omega)\}}{\text{Re}\{\text{Num}(j\omega)\}} \right) - \tan^{-1} \left( \frac{\text{Im}\{\text{Den}(j\omega)\}}{\text{Re}\{\text{Den}(j\omega)\}} \right) \text{ degrees}
\]

Step 2 – Compute the Frequency, Magnitude, and Phase Angle at the Edge of System Stability:

Discussion:
Marginal stability (i.e.: the “edge of system stability”) occurs when the system gain (magnitude) is 1 and the phase angle is \( \pm 180^\circ \). The Nyquist plot often resembles a spiral on the s-plane. The point of marginal stability occurs when the Nyquist plot intersects the point \( s = -1 \) (magnitude of 1, angle of \( \pm 180^\circ \)) on the s-plane. As -1 is a real value, this intersection must occur when the imaginary part of \( G(j\omega) \) is equal to zero.

• Compute the frequency at which the system is marginally stable (the imaginary part of \( G(j\omega) \) is equal to zero), \( \omega_0 \).

• Substitute \( \omega_0 \) into the magnitude and phase angle equations to compute the expected magnitude at \( \omega_0 \), \( M_0 = |T(j\omega_0)| \), and the associated phase angle, \( \theta_0 \).

Expected Frequency for Marginal Stability, and associated Magnitude and Phase Angle:

<table>
<thead>
<tr>
<th>( \omega_0 )</th>
<th>rad/sec</th>
<th>Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_0 )</td>
<td>(raw value)</td>
<td>dB</td>
</tr>
<tr>
<td>( \theta_0 )</td>
<td>degrees</td>
<td></td>
</tr>
</tbody>
</table>

Step 3 – Derive the Adjustment Function, \( T_{pt}(s) \), for Conversion of \( V_t(s) \) to \( V_p(s) \): Discussion:
As this (open-loop) system is not controlling the closed-loop position, the motor position may rotate more than \( 360^\circ \). If the position indicator output, \( V_p \), is being monitored, the potentiometer wiper may pass through the sensor “dead zone” during which the position indicator output transitions from +15V to -15V; this will complicate the recording of \( V_p \) data. It may therefore be necessary to “reconstruct” the \( V_p \) signal by reading the tachometer output, \( V_t \), and working through block-diagram algebra to convert \( V_t \) to \( V_p \).
• From the block diagram shown in Figure 28, derive the adjustment function, \( T_{pt}(s) \), necessary to convert the signal \( V_i(s) \) to \( V_p(s) \). The system with “reconstructed” \( V_p \) output is shown in Figure 30.

![Figure 30 - Open-Loop Third Order System showing Adjustment Function](image)

**Procedure:**

**Preliminary Preparation**

Step 1 – Perform the **Balance Pre-Amp Output Procedure** (refer to Procedure section, page 31)

Step 2 – Perform the **Zero Op-Amp Output Procedure** (refer to Procedure section, page 43)

**Closed-Loop Bode Analysis**

Step 1 – Set the gain, \( K_a \), to the value computed in Lab Exercise #2 for a 25% overshoot:

- Use the **Set Gain using an Attenuator Procedure** (refer to Procedure section, page 44)

Step 2 – Connect the system components as shown in Figure 10 (Lab Exercise #2, page 36):

- The Op-Amp feedback network is set to the 100K\( \Omega \) resistor.

Step 3 – Function Generator setup:

- Set the function generator for Sine-wave output.
- The frequency will be varied over a range of at least \( 0.1\cdot\omega_n \) to \( 10\cdot\omega_n \). The lowest frequency should be about 0.4 Hz or lower; the highest frequency should be about 2 Hz or higher.
- Set the amplitude to 10 Vpp.
- Connect the function generator output to the Op-Amp input, as shown in Figure 10.
- Confirm that the \( V_{Motor} \) input to the servo-amp is limited to \( \pm V_{MAX} \) Vpp; reduce \( V_{in} \) if necessary.

Step 4 – Measure and Record \( V_{in} \) and \( V_{out} \):

- Use either the “Procedure: Measure Sine-Waveform Phase Delay Angle” (page 32), or “Lab Exercise 7 – Appendix – The Lissajous Curve” (page 91).

Note that it may be easier to use one technique over the other at different frequency ranges; it is very difficult to read the Lissajous curve for small phase differences.
• Vary the frequency by 0.02-1 Hz between recorded samples; smaller increments if the output is changing significantly, and larger increments if the output is not changing significantly.

• Take enough samples to identify the basic shape of the Bode plot. The expected shape of the Bode plot for a second-order system with 25% overshoot can be found in the reference reading material.

• Use the Hard Copy feature on the oscilloscope to obtain several representative print-outs of the traces. Indicate the key information on the hard copy/print-out. It is NOT necessary to obtain a print-out of each sample point.

• The resonant peak magnitude may not occur precisely at the computed value of \( \omega_r \). Therefore, review the Vout magnitude data recorded and identify the range of frequencies in which the peak magnitude occurs. Then, vary the frequency in this range to **find and record the data at the resonant peak magnitude**. Compare the resonant peak magnitude data with the expected values.

• Generate the Bode Plot from the sampled data. Plot the Gain and Phase with respect to angular frequency, \( \omega \); use a logarithmic scale for \( \omega \).

<table>
<thead>
<tr>
<th>Input:</th>
<th>Set:</th>
<th>Measure:</th>
<th>Compute:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega ) (rad/sec)</td>
<td>( f ) (Hz)</td>
<td>Vin (Vpp)</td>
<td>Vout (Vpp)</td>
</tr>
<tr>
<td>rad/sec</td>
<td>Hz</td>
<td>Vpp</td>
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<tr>
<td>rad/sec</td>
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<tr>
<td>rad/sec</td>
<td>Hz</td>
<td>Vpp</td>
<td>Vpp</td>
</tr>
</tbody>
</table>

**Peak Output:**

Table 4 - Closed-Loop System Frequency Response

<table>
<thead>
<tr>
<th>Expected:</th>
<th>Measured:</th>
<th>Analyze:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_r )</td>
<td>rad/sec</td>
<td>rad/sec</td>
</tr>
<tr>
<td>( M_r )</td>
<td>dB</td>
<td>dB</td>
</tr>
<tr>
<td>( \theta_r )</td>
<td>degrees</td>
<td>degrees</td>
</tr>
</tbody>
</table>
Open-Loop Nyquist Analysis

Step 1 – Double-Check the Pre-Amp and Op-Amp Balanced/Zeroed Output:
- Perform the Balance Pre-Amp Output Procedure (refer to Procedure section, page 31)
- Perform the Zero Op-Amp Output Procedure (refer to Procedure section, page 43)

Step 2 – Set the gain, $K_a$, to the value measured in Lab Exercise #3 to achieve marginal system stability:
- Use the Set Gain using an Attenuator Procedure (refer to Procedure section, page 44)

Step 3 – Connect the system components as shown in Figure 29 (page 82):
- The Op-Amp feedback network is set to the RC network ($\tau = 0.1$ sec).
- Note that there is NO FEEDBACK from the output of the position indicator to the Op-Amp input.

Step 4 – Function Generator setup:
- Set the function generator for Sine-wave output.
- The frequency will be varied over a range of at least $0.1 \cdot \omega_0$ to $10 \cdot \omega_0$.
- Set the amplitude to 10 $V_{pp}$.
- Connect the function generator output to the Op-Amp input, as shown in Figure 29.
- Confirm that the $V_{Motor}$ input to the servo-amp is limited to $\pm V_{MAX} V_{pp}$; reduce $V_{in}$ if necessary.

Step 5 – Measure and Record $V_{in}$ and $V_{out}$:
- Use either the “Procedure: Measure Sine-Waveform Phase Delay Angle” (page 32), or “Lab Exercise 7 – Appendix – The Lissajous Curve” (page 91).
  Note that it may be easier to use one technique over the other at different frequency ranges; it is very difficult to read the Lissajous curve for small phase differences.
- Vary the frequency by 0.02-1 Hz between recorded samples; smaller increments if the output is changing significantly, and larger increments if the output is not changing significantly.
- If “reconstructing” the $V_p$ output from the $V_i$ data, be very cautious to record the correct magnitude and phase angle between $V_{in}(s)$, $V_i(s)$, and $V_p(s)$.
- Take enough samples to identify the basic shape of the Nyquist plot. The expected shape of the Nyquist plot for an open-loop third-order system can be found in the reference reading material.
- Use the Hard Copy feature on the oscilloscope to obtain several representative print-outs of the traces. Indicate the key information on the hard copy/print-out. It is NOT necessary to obtain a print-out of each sample point.
The point of marginal stability may not occur precisely at the computed value of $\omega_0$. Therefore, review the overall system magnitude and phase data recorded and identify the range of frequencies where the gain is $\sim 1$ and the phase is $\sim -180^\circ$. Then, vary the frequency in this range to **find and record the data at the point of marginal stability**.

Compare the marginal stability data with the expected values.

- Generate the Nyquist Plot. Plot the Gain and Phase on the s-plane as a polar plot.

<table>
<thead>
<tr>
<th>Input:</th>
<th>Set:</th>
<th>Measure:</th>
<th>Compute:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega$ (rad/sec)</td>
<td>$f$ (Hz)</td>
<td>$V_{in}$ (Vpp)</td>
<td>$V_{out}$ (Vpp)</td>
</tr>
<tr>
<td>$\Omega$ (rad/sec)</td>
<td>Hz</td>
<td>$V_{in}$ (Vpp)</td>
<td>$V_{out}$ (Vpp)</td>
</tr>
<tr>
<td>$\Omega$ (rad/sec)</td>
<td>Hz</td>
<td>$V_{in}$ (Vpp)</td>
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<td>$\Omega$ (rad/sec)</td>
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<td>$\Omega$ (rad/sec)</td>
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<td>$\Omega$ (rad/sec)</td>
<td>Hz</td>
<td>$V_{in}$ (Vpp)</td>
<td>$V_{out}$ (Vpp)</td>
</tr>
</tbody>
</table>

**Marginal Stability:**

<table>
<thead>
<tr>
<th>Expected:</th>
<th>Measured:</th>
<th>Analyze:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_0$</td>
<td>rad/sec</td>
<td>rad/sec</td>
</tr>
<tr>
<td>$M_0$</td>
<td>dB</td>
<td>dB</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>degrees</td>
<td>degrees</td>
</tr>
</tbody>
</table>

**Table 5 - Open-Loop System Frequency Response**
Summary:
Lab Exercise Checklist – Be sure that the following have been obtained in the Lab to complete the lab exercise:
- Collected data supporting the measurements of \( \omega_r, M_r \) and \( \theta_r \) for the Closed-Loop Bode analysis, and \( \omega_0, M_0 \) and \( \theta_0 \) for the Open-Loop Nyquist analysis.
- Representative oscilloscope traces (annotated) from both Bode and Nyquist sections; label the traces with appropriate names and sample information.

Lab Report Checklist – The Lab Report should contain the following supporting documentation:
- All collected data and hardcopies of scope traces
- Derivation of transfer functions
- Pre-lab computations including all expected values, derivation of frequency for peak magnitude, derivation of frequency for marginal stability
- Error analysis of the measured vs. expected values of \( \omega_r, M_r, \theta_r, \omega_0, M_0, \) and \( \theta_0 \)

Review Questions – Answers must be included in the lab report:

•
Lab Exercise 7 – Appendix – The Lissajous Curve

Purpose:
The Lissajous curve plots two functions against each other without an explicit time axis; time occurs implicitly. The Lissajous curve is essentially a mapping from the input waveform to the output waveform. This is a type of transfer function. Consider Figure 31.

The Lissajous curve is generated by using the X-Y display mode on the oscilloscope time base selection. The horizontal extent of the Lissajous Curve is the $V_{in}(t)$ peak-to-peak value; the vertical extent of the curve is the $V_{out}(t)$ peak-to-peak value. Centering the curve in the oscilloscope screen, the height $Q$ can be found.

This technique may be used to determine the gain and phase angle relationship between the input and output waveforms as an alternative method to comparing the two sinusoidal waveforms directly. Furthermore, measuring the phase delay (phase shift) by comparing two sinusoidal waveforms may be difficult when the time delay is very small compared to the period of the waveform. This technique may be used to facilitate the measurements in that case.

Figure 31 - Lissajous Curve Analysis
References:
Lissajous Figures
http://surendranath.tripod.com/Lissajous/Lissajous.html

ExploreLearning – Lissajous Figures “Gizmo”

Procedure:
Step 1 – Oscilloscope setup:
- Set the display mode to X-Y.
- Ground both Channel 1 (CH1) and Channel 2 (CH2) inputs.
- Center the dot vertically with the CH1 offset adjustment, and center it horizontally with the time base adjustment.
- Connect the input signal, V_{in}(t), to CH1 and the output signal, V_{out}(t), to CH2.

Step 2 – Measure V_{in}(t), V_{out}(t), and Q (all are in units of peak-to-peak voltage, V_{pp})

Step 3 – Compute Gain and Phase Angle:
- Gain is calculated as:
  \[
  \text{Gain} = 20 \cdot \log_{10} \left( \frac{V_{out-pp}}{V_{in-pp}} \right) dB
  \]
- Phase Angle is calculated as:
  \[
  \text{Phase Angle} = \Theta = \sin^{-1} \left( \frac{Q_{pp}}{V_{out-pp}} \right) \text{ degrees}
  \]

The sign of the Phase Angle is negative if the output waveform Lags the input waveform (Lissajous curve “leans” toward the first quadrant of the s-plane), and is positive if the output waveform Leads the input waveform (Lissajous curve “leans” toward the fourth quadrant of the s-plane)

Example:
The Lissajous curve in Figure 31 shows input and output waveforms, V_{in}(t) and V_{out}(t):
\[
V_{in}(t) = \sin(2\pi \cdot t)V_{pp}
\]
\[
V_{out}(t) = \frac{3}{4} \cdot \sin(2\pi \cdot t + \frac{\pi}{6})V_{pp}
\]

The values measured are:
- V_{in}(t) = 2 V_{pp}
- V_{out}(t) = 1.5 V_{pp}
- Q = 0.75 V_{pp}
The computed values are:

\[ Gain = 20 \cdot \log_{10} \left( \frac{1.5 V_{pp}}{2 V_{pp}} \right) dB = -2.5 dB \]

\[ Phase \ Angle = \sin^{-1} \left( \frac{0.75 V_{pp}}{1.5 V_{pp}} \right) \text{ degrees} = 30^\circ = \frac{\pi}{6} \text{ radians} \]

Note that Figure 31 shows an example of an output waveform that \textit{Leads} the input waveform, and therefore the Phase Angle is positive.
Appendix – TDS3012 Oscilloscope Specifications

This section written by Dr. Joseph R. Asik

Basic Description

Bandwidth 100 MHz; Sample rate 1.25 GS/s; 2 Channels

This section contains a basic description of the features and operation of the Tektronix 3012 Oscilloscope shown above. Additional details can be found in the Detailed Description.
Using the Menu System - Front-Panel Menus and Controls

The front panel has buttons and controls for the functions you use most often. The front panel has menus to access more specialized functions. To use the menu system, follow the steps shown below.

1. Push a dark-colored front-panel menu button to display the menu you want to use.

![Front-panel menu button image]

2. Push a bottom screen button to select a menu item. If a pop-up menu appears, continue to push the screen button to select an item from the pop-up menu.

![Bottom screen button image]
3. Push a side screen button to choose a menu item. If the menu item contains more than one choice, push the side screen button again to make the choice.

4. Certain menu choices require you to set a numerical value to complete the setup. Use the general-purpose knob to adjust the parameter value. Push the COARSE button to make larger adjustments.

Using the Menu Buttons

You can use the menu buttons to perform the following oscilloscope functions.

1. MEASURE. Performs automated measurements of waveforms.

2. CURSOR. Activates the cursors.

3. SAVE/RECALL. Saves and recalls setups and waveforms to memory or a floppy disk.
Saving to disk allows the student to transfer data electronically to his/her report.

4. DISPLAY Changes the appearance of waveforms and the display screen.

5. QUICKMENU. Activates QuickMenus such as the built-in Scope QuickMenu.

6. UTILITY Activates various system utility functions, such as selecting a language.
7. Vertical MENU. Adjusts the scale, position, and offset of waveforms. Sets the input parameters.

8. Trigger MENU. Adjusts the trigger functions.

9. Acquire MENU. Sets the acquisition modes and horizontal resolution, and resets the delay time.

**Using the Dedicated Controls**

These dedicated buttons and controls generally control waveforms and cursors without the use of menus.

1. COARSE. Causes the general-purpose knob and position knobs to make adjustments more quickly.

2. SELECT. Toggles between the two cursors to select the active cursor.

4. Vertical POSITION. Adjusts the vertical position of the selected waveform. Push COARSE to make adjustments more quickly.

5. Horizontal POSITION. Adjusts the trigger point location relative to the acquired waveforms. Push COARSE to make adjustments quickly.

6. Trigger LEVEL. Adjusts the trigger level.

7. RUN/STOP Stops and restarts acquisition.

8. SINGLE SEQ. Sets acquisition, display, and trigger parameters for a single-shot (single sequence) acquisition.

9. SET TO 50%. Sets the trigger level to the midpoint of the waveform.

10. AUTOSET. Automatically sets the vertical, horizontal, and trigger controls for a usable display.

11. FORCE TRIG. Forces an immediate trigger event.

12. WAVEFORM INTENSITY Controls waveform intensity.

13. B TRIG. Activates the B Digger. Changes the trigger menu to set the B-trigger parameters.

14. DELAY Enables delayed acquisition relative to the trigger event. Use horizontal POSITION to set the amount of delay.

15. Horizontal SCALE. Adjusts the horizontal scale factor.

16. Horizontal zoom. Splits the screen and magnifies the current acquisition horizontally.
17. Waveform OFF. Removes selected waveform from the display.

18. Vertical SCALE. Adjusts selected waveform vertical scale factor.

19. CH1, CH2 MATH. Displays a waveform and chooses the selected waveform. REF shows the reference waveform menu.

20. Hard copy. Initiates a hard copy from the dot matrix printer using the port selected in the Utility menu.

21. Power switch. Turns power to on or standby. Power-up time varies from about 15 seconds to 45 seconds, depending on the oscilloscope internal calibration process.

22. Wrist-strap ground. Connect a wrist strap when working with ESD-sensitive circuits. This connector is not a safety ground.

23. MENU OFF Clears menu from the display.

**Hard Copy**

After you connect a printer and set up the oscilloscope, push the hard-copy button at the left of the display to make a hard copy. You can also store hard-copy images on a floppy disk (in normal or compressed format) and then transfer them later to a PC for printing or use in a report.

**Connecting a Printer**

Connect your printer to the parallel printer port on the rear panel of the oscilloscope.

**Setting Up to Print**
Follow these steps to set up the oscilloscope to print a hard copy:

1. Push the **UTILITY** menu button.

2. Push the System screen button to select **Hard Copy**.

3. Push the Format screen button and then choose the printer format appropriate for your application.

4. Push the **Options** screen button to select the image orientation (portrait or landscape) as well as turn on or off hard copy file compression.

5. Push the **Ink Saver** screen button and select **On** for most applications. If you want the hard copy colors to be the same as the screen colors, you can select **Off**.

6. Push the **Port** screen button and select the port that your printer is connected to or select **File** to save the hard copy on a floppy disk.

7. Push the hard copy button.

**Hard Copy File Compression.** When compression is set to on, the oscilloscope compresses the hard copy data, using the current printer format, into a gzip file format, with the extension .gz. Compressing hard copy files lets you store more screen captures on a floppy disk. Centronics output is never compressed. .gz files can be decompressed using PKZIP™ or WinZip™ programs.

**Color and Gray-Scale Printing.** You can print a color hard copy that uses the display colors. Gray-scale waveform information is printed as shades of color. If you have a Deskjet or Laserjet monochrome printer, gray-scale waveform information is printed as a dithered image.

**Ink Saver and Preview.** As an alternative to printing the display colors, turn on the Ink Saver function to print a hard copy with a white background. This function saves printer ink while it preserves the color coding of the waveforms and readouts. Ink saver also works with the monochrome print formats.

Push and hold the **Preview** screen button to show how the colors will appear on the paper.

**Clear Spool.** You can push the Clear Spool screen button to empty the print spooler to stop a hard copy operation in progress, if the hard copy port connection is not made due to incompatible settings (such as baud rate), or if you lose the hard copy port connection before the hard copy is complete.
Tektronix 3012 Oscilloscope – Detailed Description

Identifying Items in the Display

The following items may appear in the display; not all items are visible at any given time. Some readouts move outside the graticule area when menus are turned off.

1. Waveform baseline icons show the zero-volt level of the waveforms (ignoring the effect of offset). The icon colors correspond to the waveform colors.

2. Acquisition readout shows when acquisition is running, stopped, or when acquisition preview is in effect.

3. Trigger position icon shows the trigger location in the waveforms.

4. Expansion point icon shows the point that the horizontal scale expands and compresses around.

5. Waveform record icon shows the trigger location relative to the waveform record. The line color corresponds to the selected waveform color.

6. Trigger status readout show trigger status.
7. Trigger level icon shows the trigger level on the waveform. The icon color corresponds to the trigger source channel color.

8. Cursor and measurement readouts show results and messages.

9. Trigger readouts show the trigger sources, slopes, and levels, and position.

10. Readout shows the delay setting or the trigger location within the record.

11. Horizontal readout shows the main or zoom time/division.

12. Auxiliary waveform readouts show the vertical and horizontal scale factors of the math or reference waveforms.

13. Channel readouts show the channel scale factor, coupling, input resistance, bandwidth limit, and invert status.

14. Triangle icon with the battery icon indicates a battery is installed and battery power is in use. The battery icon shows the approximate charge level of the battery.

15. Power-plug icon with the battery icon indicates a battery is installed but line power is in use. The battery may be charging. The battery icon shows the approximate charge level.

**Using QuickMenus**

The QuickMenu feature simplifies the use of the oscilloscope. When you push the QUICKMENU button, a set of frequently used menu functions show on the display. Then, push the screen buttons around the display to operate the QuickMenu.
Using the Scope QuickMenu. Scope is one type of QuickMenu that you can use to control the basic oscilloscope functions. You can perform many tasks without using the regular menu system. If you need to use a function that is not contained in the Scope QuickMenu, push the button you would normally push to access that function. For example, if you want to add an automatic measurement, push the MEASURE button to set up the measurement. Then, push the QUICKMENU button to return to the Scope QuickMenu with the measurement also in the display.

1. Edge Trigger controls. Push these screen buttons to set trigger parameters for edge trigger.

2. Trigger controls if either B trigger or video trigger is selected.

3. Cursor control. Push this screen button to turn on cursors and select the cursor type. Push the SELECT button to toggle between the two cursors to select the active cursor. Use the general-purpose knob to move the active cursor.

4. Acquisition controls. Push these screen buttons to set acquisition parameters.

5. Channel vertical controls. Push these screen buttons to set vertical controls for the selected channel. Use the CH 1, CH2, CH3, CH4, MATH, and REF buttons to select the channel you want to control.
6. Vertical controls if either the math waveform or a reference waveform is selected.

7. Menu. Push this screen button to select a specific QuickMenu display if more than one is available.

**NOTE.** Items in the Scope QuickMenu not mentioned above are also contained in the regular display.

**Other QuickMenus.** Some optional application packages include a custom QuickMenu display. Those QuickMenus contain specific features that are important for the application.

---

**Front-Panel Connectors**

1. **PROBE COMP** Square wave signal source to compensated probes.

2. **CH 1 & CH 2.** Channel inputs with TekProbe interface.

3. **EXT TRIG.** External trigger input with TekProbe interface (two-channel models only).
**Rear-Panel Connectors**

1. Parallel printer port. Connect to a printer to make hard copies.

2. CAL switch. For use by authorized service personnel only.

3. DC power output. Provides -15 V DC accessory power only when the oscilloscope is connected to the AC power line.

4. Ground terminal. Connect to earth ground when using battery power.

5. Power input. Attach to an AC power line with integral safety ground.

6. GPIB port. Connect to a controller for remote programmability.

7. RS-232 port. Connect to a controller or terminal for remote programmability or printing.

8. VGA port. Connect to a VGA monitor to display the screen image.

9. 10baseT local area network (LAN) Ethernet port. Connect to a 10baseT network for remote printing or programming.

**Application Examples**

This section presents two common oscilloscope applications:

- 1. Taking simple measurements
- 2. Analyzing signal detail
Each application example highlights different features of the oscilloscope and gives you ideas about using the oscilloscope to solve test problems.

**Taking Simple Measurements**

You need to see a signal in a circuit, but you do not know the signal amplitude or frequency. Connect the oscilloscope to quickly display the signal and then measure its frequency and peak-to-peak amplitude.

**Using Autoset**

To quickly display a signal, do these steps:

1. Connect the channel 1 probe to the signal.

2. Push the **AUTOSET** button.

The oscilloscope sets vertical, horizontal, and trigger controls automatically. You can manually adjust any of these controls if you need to optimize the display of the waveform.

When you are using more than one channel, the autoset function sets the vertical controls for each channel and uses the lowest-numbered active channel to set the horizontal and trigger controls.

**Selecting Automatic Measurements**

The oscilloscope can take automatic measurements of most displayed signals. To measure signal frequency and peak-to-peak amplitude, do these steps:

1. Push the **MEASURE** button to see the measurement menu.
2. Push the CH 1 button and then push the Select Measurement for Chl screen button.
3. Select the Frequency measurement.
4. Push the more screen button until you can select the Pk-Pk measurement.
5. Push the MENU OFF button.

The measurements show on the screen and update as the signal changes.

### Measuring Two Signals

You are testing a piece of equipment and need to measure the gain of its audio amplifier. You have an audio generator that can inject a test signal at the amplifier input. Connect two oscilloscope channels to the amplifier input and output as shown. Measure both signal levels and use these measurements to calculate the gain.
To display the signals connected to channels 1 and 2, do these steps:

1. Push the CH 1 and CH 2 buttons to activate both channels.

2. Push the AUTOSET button.

To select measurements for the two channels, do these steps:

1. Push the MEASURE button to see the measurement menu.

2. Push the CH 1 button and then push the Select Measurement for Ch1 screen button.

3. Select the Amplitude measurement.

4. Push the CH 2 button and then push the Select Measurement for Ch2 screen button.

5. Select the Amplitude measurement.

6. Calculate the amplifier gain using the following equations:

\[
Gain = \frac{\text{output amplitude}}{\text{Input amplitude}} = \frac{3.155 \text{ V}}{130.0 \text{ mV}} = 24.27
\]

\[
Gain (\text{dB}) = 20 \times \log(24.27) = 27.7 \text{ dB}
\]
Customizing Your Measurements

In this example you want to verify that the incoming signal to a piece of digital equipment meets its specifications. Specifically, the transition time from a low logic level (0.8 V) to a high logic level (2.0 V) must be 10 ns or less.

To select the rise time measurement, do these steps:

1. Push the MEASURE button to see the measurement menu.

2. Push the CH 1 button and then the Select Measurement for Chl screen button.

3. Select the Rise Time measurement. Rise time is typically measured between the 10% and 90% amplitude levels of a signal; these are the default reference levels the oscilloscope uses for rise time measurements. However, in this example you need to measure the time the signal takes to pass between the 0.8 V and 2.0 V levels.

You can customize the rise time measurement to measure the signal transition time between any two reference levels. You can set each of those reference levels to a specific percent of the signal amplitude or to a specific level in vertical units (such as volts or amperes).

Setting Reference Levels. To set the reference levels to specific voltages, do these steps:

1. Push the Reference Levels screen button.

2. Push the Set Levels in screen button to select units.

3. Push the High Ref screen button.

4. Use the general-purpose knob to select 2.0 V.

5. Push the Low Ref screen button.

6. Use the general-purpose knob to select 800 mV

The measurement verifies that the transition time (3.842 ns) meets the specification (10 ns).
Measuring Specific Events

Next you want to see the pulses in the incoming digital signal, but the pulse widths vary so it is hard to establish a stable trigger. To look at a snapshot of the digital signal, do this step:

1. Push the SINGLE SEQ button to capture a single acquisition.

Now you want to measure the width of each displayed pulse. You can use measurement gating to select a specific pulse to measure. To measure the second pulse, for example, do these steps:

1. Push the MEASURE button.
2. Push the CH 1 button and then push the Select Measurement for Chl screen button.
3. Select the Positive Width measurement.
4. Push the Gating screen button.
5. Select Gate With V Bar Cursors to choose measurement gating using cursors.
6. Place one cursor to the left and one cursor to the right of the second pulse.

The oscilloscope shows the width measurement (160 ns) for the second pulse.
Taking Cursor Measurements

You can use the cursors to take quick measurements on a waveform. To measure the ring frequency at the rising edge of the signal, do these steps:

1. Push the CURSOR button.
2. Push the Function screen button.
4. Push the V Bar Units screen button.
5. Select 1/seconds (Hz).
6. Place one cursor on the first peak of the ring using the general-purpose knob.
7. Push the SELECT button.
8. Place the other cursor on the next peak of the ring.
The cursor Δ readout shows the measured ring frequency is 227 kHz.

![Graph Image]

**Acquisition Features**

**Separate Digitizers.** Ensure accurate timing measurements with separate digitizers for each channel. Each digitizer can sample at up to the maximum sample rate; acquisition on all channels is always concurrent to provide full single-shot bandwidth on each channel.

**Normal Acquisition.** Acquire 10,000-point waveforms to capture horizontal detail and then use the zoom @function to analyze the detail.

**Fast Trigger Acquisition.** Acquire up to 3,000 waveforms per second to see rapidly changing signals or intermittent signal irregularities.

**Pretrigger.** You can capture signals that occur before the trigger point. You can position the trigger point at the beginning of the acquisition, at the end, or at any location in between.

**Delay.** You can also delay the acquisition so that it starts after the trigger point. Use delay when you want to acquire the signal at a specific time after the trigger point.

**Peak Detect.** See pulses as narrow as 1 ns even at the slower time base settings. Peak Detect helps you see noise and glitches in your signal.
Signal Processing Features

**Average.** Apply averaging to your signal to remove uncorrelated noise and improve measurement accuracy.

**Envelope.** Use envelope to capture and display the maximum variation of a signal.

**Waveform Math.** Use waveform math to add, subtract, multiply, or divide waveforms. For example, you can use math to analyze differential signals or calculate a power waveform.

Display Features

**Color LCD Display.** Identify and differentiate waveforms easily with color-coding. Waveforms, readouts, and buttons are color matched to increase productivity and reduce operating errors.

**Digital Phosphor.** A Digital Phosphor Oscilloscope can clearly display intensity modulation in your signals. The oscilloscope automatically overlays subsequent acquisitions and then decays them to simulate the writing and decay of the phosphor in an analog oscilloscope CRT. This feature results in an intensity-graded waveform display that shows the information contained in the intensity modulation.

**Signal Preview.** Use the preview feature to optimize the control settings when setting up a single-shot acquisition. As you adjust the controls, the adjustments modify the current acquisition to show a preview of how the next acquisition should appear.

Measurement Features

**Cursors.** Use cursors to take simple voltage, time, and frequency measurements.

**Automatic Measurements.** Choose from a list of 21 automatic waveform measurements. You can customize the measurements by changing reference levels or by adding measurement gating.

Trigger Features

**Dual Triggers.** Use the main (A) trigger system alone or add the B trigger to capture more complex events. You can use the A and B triggers together to set up a wait-for-time or wait-for-events trigger.

**Video Trigger.** Trigger on video fields or lines to see a stable display of standard video signals.
**Convenience Features**

**Autoset.** Use Autoset to quickly set up the vertical, horizontal, and trigger controls for a usable display.

**Scope QuickMenu.** Use the built-in Scope QuickMenu for simplified oscilloscope operation.

**Single Sequence.** One button sets the trigger parameters to the correct settings for a single-shot acquisition (or single-sequence acquisition).

**Floppy Disk.** Use the built-in floppy disk to store and recall waveforms and setups, as well as upgrade the oscilloscope firmware and install new features.

**Probe Support.** Use the standard probes or choose an optional probe for a specific application.

**Optional Features**

**Communication Modules.** Install a communication module to add RS-232, GPIB, VGA, or Ethernet local area network (LAN) ports for remote programmability, sending hardcopy to a LAN printer, or displaying the oscilloscope screen on a monitor.