Course Syllabus

Topic: Power in AC circuits, magnitude and phase, real and reactive power, complex power, apparent power, power triangle, leading and lagging power factor, Watt meters, power factor correction. (2 weeks)

Read: O'Malley Chapter 15, 2_Power.mw, and Johnston Chapter 1
Problems: O'Malley Chap. 15 - 43, 45, 46, 47, 48, 49, 53, 54, 55, 58, 63, 64, 66, 68, 70, 73, 74, 75, 78, 80, 83 There is a mistake in the answer to 15.43 it should read, “Ans. P_max = 10W, P_min = 2W, P = 6W”

Johnston Chap. 1 Problems 1, 2, 3, 4. (Additional Single Phase Power Problems)

Topic: Transformers: single phase transformers, polarity, dot convention open circuit test, short circuit test, calculation of the Steinmetz coefficients, voltage regulation, efficiency, multi winding transformers, reflected impedance, auto-transformers. (2 weeks)

Read: 3_Transformers.mw, Sen Chap. 2, and Johnston Chap. 5
Problems: Sen Chap. 2 - 1, 3, 4, 6, 7, 8, 9, 10, 15, 16, 17 O'Mally 16.55, 56, 57, 60, 61

Topic: Three Phase Systems: subscripts, three-phase voltage generation, phase sequence, Y connected loads, Delta connected loads, three-phase power, unbalanced three-phase circuits. (2 Weeks)

Read: 4_ThreePhase.mw, O'Malley Chapter 17 and Johnston Chapter 2
Problems: O'Malley Chap. 17 - 51, 53, 56, 57, 58, 61, 63, 65, 68, 70, 71, 77, 79, 80, 82, 83, 98, 99
Johnston Chap. 2 - 1 (Three Phase Supplementary Problem)

Topic: Three-phase transformers, Y-Y, Y-Delta, Delta-Y, V-Delta, and Delta-Delta connected transformers, single-phase equivalents, harmonics, per-unit system. (1 weeks)

Read: Sen Chapter 2 and Johnston Chapter 6
Problems: Sen Chap. 2, 19, 20, 21, 23

Topic: Magnetic Circuits: I-H relation B-H relation, equivalent circuits, load-lines, magnetization curve, air gaps, inductance, hysteresis, hysteresis losses, eddy current losses, sinusoidal excitation, permanent magnets, permanent magnet materials, residual flux, coercive force shear line, recoil line. (2 Weeks)

Read: 5_MagneticPhenomena.pdf, 6_Magekts.mw, Sen Chap. 1 and Johnston Chapter 3 and Chapter 4
Problems: Sen Chap. 1 - 1, 4, 5, 6, 7, 9, 14, 15, 16, 19, 24, 25, 26
Johnston Chap. 4 - 1, 2, 3, 4 (Num. Approaches to Mag. Flux HW Probs.)

Topic: Transformer Design: cores, window area, voltage, current, and power relations, induced emf, saturation, constant current density, equal area criterion, fill factor, area product, heat dissipation, radiation, convection, copper losses, mean turn length, transformer design equations, American wire gauge, cmils, National Electric Code, other core styles (2 Weeks)

Read: 7_xfmrdsgn.mw, Johnston Chapter 7
Problems: Johnston Chap. 7 - 1, 2

Topic: Electromechanical Energy Conversion: field energy, co-energy, mechanical force linear electromechanical systems, rotating machines, DC machines, power, efficiency. (2 Weeks)

Read: Sen Chapter 3 and Johnston Chapter 8
Problems: Sen Chap. 3 - 1, 2, 3, 4, 5, 6, 7
EEE 3511 Introduction to Electrical Systems Lab Syllabus

Experiment #1 (Two weeks)
Experiment #3 (Two weeks)
Experiment #5 (Three weeks)
Midterm Exam (One week)
Experiment #2 (Three weeks)
Experiment #6 (Two weeks)
Experiment #8 (Two weeks)
Final Exam (One week)
1. The Electric Machines Laboratory (Room E-19) is equipped with several safety features.
   a. The room power kill switch is a large red mushroom button located on a steel column near center of the room. Pressing the red button disconnects power to all the laboratory stations. Room lighting is not affected. If any problem is perceived at your own or any other lab station, any student can and should push red power kill switch.
   b. Located on the same column is a telephone with an outside line. In an emergency, you should dial *911. You will be connected to the emergency desk of LTU security, who will arrange the necessary emergency response. This information is also posted on the phone.
   c. Fire extinguishing equipment is located on the north wall, just to the right of the door.
   d. A first aid kit is located on the west wall, just to the left of the door.

2. General housekeeping rules include the following:
   a. No smoking in the laboratory
   b. No food or drinks at the lab benches
   c. Pick up and dispose of trash in the receptacles
   d. Return all equipment to proper locations. Store test leads in box provided at each station. Leave the station in a way you would like to find it on your return.

3. Eye protection is required while the station is energized. It is intended to protect eyes from mechanical hazards of rotating equipment and sparks and molten metal in the event of a power fault. Government safety standards (Federal O.S.H.A. regulations and Michigan M.I.O.S.H.A.) rules require protective eye shielding be used. These standards apply to school as well as any facility you may work at in the future. We strive to develop healthy safety practices in our laboratory to prepare you for a safe career.
   a. Safety glasses are provided. Many students ask if they can wear their own safety glasses from outside jobs. This is permissible as long as the glasses are true safety glasses.
b. True safety glasses have the Z-87 on the frame indicating compliance with the ANSI Z-87 standard. The lenses have an etch mark identifying the manufacturer.

4. Long hair, neck ties, and jewelry are safety hazards in the vicinity of rotating equipment.
   a. Remove ties or tuck into shirt.
   b. Fix hair in such a fashion as to prevent entanglement in machines, or tuck down the back of a lab coat. (Lab coats are not provided.)
   c. Remove all rings, watches, and dangling earrings.

5. A Safety Officer should be selected for each station. The Safety Officer will be noted on each student's lab procedure. This job should rotate from week to week. The Safety Officer is responsible for ensuring that some one is worrying about safety. The Safety Officer must:
   a. Verify that the main breaker of the lab station is off before the instructor energizes the laboratory stations.
   b. Give the other students permission to energize the main and all other breakers at the station after he/she determines that everyone is prepared and safe. (Any student may turn off a breaker at any time, but the Safety Officer must give permission to re-energize). Example communication: "May I energize the A.C. breaker?" followed by "Clear to energize A.C. breaker."
   c. Verify that the rules enumerated in 4. are followed.

[The Safety Officer in an industrial job would hang tags on all switches which have been opened to provide protection. Many facilities place locks on the switches to prevent them from being operated. Only the Safety Officer who placed the tags or locks can remove the tags or locks to allow operation of a switch, after he/she has determined that all the people working under his/her protection are clear and aware of the fact that safety protection is being terminated. If several tasks are being conducted that require a certain switch to remain in position, there will be several tags or locks placed by the Safety Officers for each task. A Safety Officer can only remove his or her own tags.]
6. Safe practices

   a. Never touch an exposed conductor, or test lead end. If you always act as if exposed conductors are energized, you will be safe on the day they are accidentally energized when power is supposed to be off.

   b. Do not touch the end of a meter lead if the other meter lead is connected. Do not lay one end of a test lead down on the bench until both leads are disconnected.

   c. Always remove the power source end of a lead first.

   d. Keep as many open breakers as possible between you and the power source when working on machinery.

[Typical high voltage procedures entitle a worker to a visible open air gap between the power source and any equipment he/she must touch as well as a temporary ground connection to the circuit that must be touched. By these standards, an open circuit breaker with enclosed contacts like ours is not adequate protection. High power circuit breakers can be removed from their mounting cabinets to provide a visible air gap to a worker.]

7. Equipment Protection procedures

   a. If possible, energize equipment slowly, as switching transients can cause excessive voltages and currents. Even if the equipment is fused, it slows the class down if we must change fuses. (Our transformers are particularly prone to blowing fuses.)

   b. Start with the meters at their least sensitive range (the highest one) if you don't know how much current will flow.

   c. Do not accidentally connect an ammeter (or the ammeter terminals of a wattmeter) where you really wanted a volt meter. Ammeters (or the ammeter terminals of wattmeters) have a very low resistance in order to measure the flow of current through a wire without impeding it. They are to be connected between two points that would normally have a solid wire connection between them, instead of the wire! Improper connection of an ammeter (or the ammeter terminals of a wattmeter) will usually blow an internal fuse. Our panel meters have circuit breakers with a reset button on the panel near the readout.
d. Accidentally connecting a volt meter in place of an ammeter will usually not do any damage, but the circuit is no longer correct and the data will be useless. A volt meter is intended to measure the voltage or "electric pressure" between two points in a circuit. It has a very high resistance and thus allows very little current to flow through it, in an attempt to not disturb the circuit under test. If a volt meter is inserted in place of an ammeter, the intended current path through the intended ammeter will not exist through the voltmeter.

8. Open/Closed switch or breaker terminology often confuses new students, particularly mechanical students used to thinking in terms of fluid flow.

a. An Open switch is in the off position (usually down). There is an opening in the path of the conductor carrying current through the switch that blocks the flow of current. (The opposite of an open valve that allows fluid to flow!)

b. A closed switch is in the on position (usually up) The gap between the switch terminals is closed by a conducting switch blade that allows current to flow through the switch. (The opposite of a closed valve that does not allow fluid to flow!)

LAB POLICY

LAB ATTENDANCE AND REPORTS: Lab reports are due at the beginning of the scheduled lab period following the period in which the experiment was performed. You must attend and participate in the lab in order to submit a lab report for a grade. Late reports will be downgraded ten points (on a scale of one hundred) for each business day that they are late. If you must submit a late report when the professor is not available, then submit the report to the departmental secretary for her to note the time and date on the report. Make sure that professor's name shows on the report so it gets to the right place.

QUALITY OF LAB REPORTS: Correct grammar, punctuation, and spelling are required. Although typing is not absolutely necessary, professional neatness is required before a 100 will be given. Curves must be plotted by computer or be neatly and properly drawn on appropriate graph paper.

LAB REPORT FORMAT:

PRELAB: Before a lab experiment may be performed, a PRELAB report must be prepared and submitted to the lab instructor for his/her approval. The Prelab Report counts 20% of the total lab report. The graded Prelab Report must be included in the main lab report. The Prelab
Report should include empty data tables for recording the results of the experiments, any calculations that can be completed before the experiment is performed, and a brief statement of the purpose of the experiment.

MAIN LAB REPORT: The report must be done neatly. The preferred presentation is with Maple, Mathcad, or word processor. The advantage of using Maple or Mathcad is that the text, equations, calculations and graphs can all be handled by the same program. Handwritten reports are not accepted. Make the report as brief as possible; unnecessary filling of the report will be downgraded. However, the report is a narrative document and must stand on its own.

TITLE PAGE: The title page must have the format shown on the next page. (Photocopies of this page are acceptable.)

GRADED PRELAB: The graded prelab follows immediately after the title page.

CALCULATIONS: Show all the equations and sample calculations used to obtain the requested results. Show these calculations in the same order as requested in the experiment. Refer to the part of the experiment to which they apply. Maple, Mathcad or a spreadsheet is preferred.

RESULTS: Answer the questions and provide the graphs or data in the same order as requested by the experiment. Refer to the part of the experiment to which these apply.

GRAPHs: All axes are to be identified by the quantity tested and the units. Each graph must have a title identifying the equipment, the rating of the equipment, and the type of test.

DISCUSSION: The discussion is a brief (not step by step) description of what you did, what you found, and how what you found compares to the relevant theoretical prediction. If you believe that you encountered errors or difficulties in the experiment, explain where they occurred and how they may have influenced your results.

FAILURE TO FOLLOW THIS FORMAT WILL RESULT IN DOWNGRADING
Safety is everyone's responsibility.
Nowhere is this truer than when using electricity.

Using electricity is like swimming, in that novices and experts can enjoy its benefits provided that they both follow the rules at all times. None of us can fully depend on others to provide a safe working or playing environment. You have more control than anyone else over your activities and the caution you use when participating in those activities.

Tektronix encourages you to learn and follow these general precautions, and to read and follow instructions specific to circuits or equipment you work with.

- Never work alone. Learn first aid, especially cardio-pulmonary resuscitation (CPR), for electrical accident victims.
- Turn off power before working on electric or electronic circuits, except when absolutely necessary. Consider all wire and terminals to be live until proven otherwise by a safe test method.
- Be sure your test equipment is operating properly before you use it.
- Do not work on electronic circuits or equipment while standing on a wet floor, or when touching plumbing or metal objects that may provide a hazardous earth ground path.
- Whenever possible, make current and voltage measurements with one hand in your pocket or behind you.
- Resist the temptation to throw a switch "to see what happens."
- Turn off power and unplug equipment before checking or replacing fuses. Locate and correct the cause of a blown fuse or tripped circuit breaker before replacing the fuse or resetting the breaker.
- Replace defective cords and plugs. Form a habit of inspecting for defects such as frayed wires, loose connections, and cracked insulation.
- Remove metal jewelry, watches, rings, chains, etc., before working on electrical circuits or equipment.
- Always check the electrical ratings of equipment you use, and be sure you use that equipment within its ratings.
- In general, treat all circuits as if high voltage or current is present.
Safety Measures

Ground Wire
A primary safety measure is grounding equipment chassis through a wire in the power cord. This practice is variously referred to as “green-wire ground” because of the color of the insulation on the chassis ground wire, or “third-wire ground” because a ground wire is a third wire.

The main reason for the ground wire is to provide a path for any fault current. No fault current would flow through the user if he/she touched the chassis. If an internal electrical fault should somehow apply dangerous voltage to the chassis of an instrument with a grounded chassis, the chassis ground wire would conduct the fault voltage to ground. In the process, the current might trip a circuit breaker or blow a fuse, which would alert the user that the instrument had a problem.

Floating Measurements
For the reason explained in the ground wire information, do not cut off the ground terminals of power cords in order to make “floating measurements”—doing so defeats ground protection. Floating measurements are referenced to some voltage other than ground potential.

The market offers various products, such as the Tektronix A6902B and the A6901, that permit floating measurements.

One technique of making floating measurements is to use a buffer to isolate the device being tested from the measuring part of the test instrument. The Tektronix A6902B Voltage Isolator uses this buffer technique, which extends the range of the test instrument to 3000 V (dc + peak ac) or 500 V (dc + peak ac), depending on the type of probe used.

Another way of making floating measurements is to isolate the power supply of the test instrument from the AC power-line ground reference. The Tektronix A6901 Ground Isolation Monitor uses this method, which allows an instrument’s chassis to float up to ±40 V (28 Vrms) from ground.

Symbols Marked on Equipment

- DANGER - high voltage
- Protective ground (earth) terminal
- ATTENTION - see operator’s manual

Symbols in Manuals

- ATTENTION - This symbol indicates the location of applicable cautionary or other information in Tektronix operator’s and service manuals.

What is Electric Shock?

According to Stedman’s Medical Dictionary, electric shock is “a sudden violent impression caused by the passage of a current of electricity through any part of the body.” This says nothing about the magnitude of that current.

The human body is electrically controlled, that is, it operates in response to its own minute electrical signals. Different persons have different resistances and sensitivities to electricity.

We recognize that electric shock is voltage dependent. One does not expect to get a shock from a battery or other low-voltage source. Sources below 30 volts are usually no problem. When voltages above 30 volts are present, precautions to prevent electric shock are appropriate. We must guard against any shock that could be fatal itself, or cause a severe reaction. We even want to prevent perception of the current.

The threshold of current perception is about 0.5 mA for 99.9% of the population, according to Charles F. Dalziel. In other words, 999 persons out of a thousand will perceive a current of 0.5 mA; one will not.

Cord-connected appliances and equipment usually comply with this 0.5 mA leakage current limit. Some industrial equipment may exceed this value. Equipment with greater leakage current is usually marked “Warning—the protective grounding conductor provides protection from electric shock; this equipment must be earth grounded for adequate protection.”

The “let-go current” is the maximum current a person can tolerate (the average is 9 mA and 6 mA for men and women, respectively) and still release the conductor by using the muscles directly stimulated by the current. (From “Electric Shock Hazard,” by Charles F. Dalziel.) The “conductor” is the source of current that the subject has grasped.

“Further increase in current up to values that are not well-defined but thought to be on the order of 100 mA may cause a fibrillation of the heart,” wrote K.S. Geiges in an article entitled “Electric Shock Hazard Analysis.” In this article, Geiges specified five important parameters, as follows:

<table>
<thead>
<tr>
<th>Lowest resistance of the body</th>
<th>500 Ω</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wet Skin, outdoors</td>
<td>1500 Ω</td>
</tr>
<tr>
<td>Dry Skin, indoors</td>
<td>6 to 20 mA, ac, depending on the person.</td>
</tr>
<tr>
<td>Let-Go Current for Adults</td>
<td>5 mA @ 20 V</td>
</tr>
</tbody>
</table>

| Safe current, adults         | 5 mA @ 30 V |
|------------------------------| 5 mA @ 20 V |
"Current caused by" ordinary household voltage (120 V, North America; and 240 V, Europe) will be 240 mA (120 V/500 Ω) and 480 mA (240 V/500 Ω), showing that lethal shock can occur in the home. Or, we add, in any place where standard electricity is available.

Fuses and circuit breakers will open the circuit under fault conditions. The time it takes these protective devices to open is long compared to body reaction times and electric shocks can result. Ground Fault Current Indicators (GFCI) are devices that sense current in the hot and neutral ac power lines. If currents are unequal, the difference must be a ground current, which causes the GFCI to open the circuit. A GFCI can be used to prevent severe electric shock when the line current is diverted into a fault. With a GFCI, a reaction will occur but fibrillation should not result.

"Currents above those possible" from ordinary household voltage across a body impedance of .500 Ω (usually in the ampere range) can affect the nerve centers, causing paralysis. The most common effect of paralysis is respiratory failure. (Power linemen are subject to this.) Such current passing through the body causes hemorrhages and burns." (K.S. Geiges

Summary of Electric Shock Effects

- Currents above the reaction-current level may cause involuntary movement and trigger a serious accident.
- If long continued, currents in excess of one's let-go current passing through the chest may produce collapse, unconsciousness, asphyxia, and death.
- An alternating current of 20 μA may produce ventricular fibrillation if injected directly into the human heart. Deaths are currently ascribed to medical apparatus in which minute stray currents are alleged to cause fatalities.
- Currents in the order of milliamperes flowing through nerve centers controlling breathing may produce respiratory inhibition that may last for a considerable period, even after interruption of the current.
- Cardiac arrest may be caused by relatively low currents flowing in the region of the heart.
- Current in the order of amperes may produce fatal damage to the central nervous system.
- Electric currents may produce deep burns, and currents sufficient to raise body temperature substantially produce immediate death.

- Delayed death may result from serious burns or other complications.

Physiological Effects of Electrical Currents

<table>
<thead>
<tr>
<th>Amperes</th>
<th>Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>Threshold of sensation</td>
</tr>
<tr>
<td>0.01</td>
<td>Cannot let go</td>
</tr>
<tr>
<td></td>
<td>Pain</td>
</tr>
<tr>
<td></td>
<td>Mild sensation</td>
</tr>
<tr>
<td>0.1</td>
<td>Extreme difficulty breathing</td>
</tr>
<tr>
<td></td>
<td>Breathing upset and labored</td>
</tr>
<tr>
<td>0.2</td>
<td>Heart goes into fibrillation death may result</td>
</tr>
<tr>
<td></td>
<td>CPR &amp; additional medical help required to sustain life</td>
</tr>
<tr>
<td>1.0</td>
<td>Severe burns Heart stops</td>
</tr>
<tr>
<td></td>
<td>Death will result without help</td>
</tr>
</tbody>
</table>

CPR & additional medical help required to sustain life

Medical assessment advised


SAFETY QUIZ

1. Name four safety features of the E19 lab.

2. Do you wear glasses? _____, Do they have the ANSI marking? _____

3. Where is the power kill switch? __________________________

4. Where is the telephone? __________________________

5. Who should be called in case of emergency? __________________________

6. Does an open switch pass current? __________________________

7. What two meters can be damaged by improper connection? __________________________

8. What meter is not likely to be damaged by miss-connecting? __________________________

9. Who must give permission to energize a breaker? __________________________

10. Who may turn off a breaker or switch? __________________________

11. What is the safe current for adults? __________________________

12. At what current does the heart go into fibrillation? __________________________

13. What voltage would produce the above current for wet skin outdoors? __________________________
Each station is laid out as shown below. The internal connections of each supply are shown on the following pages.

<table>
<thead>
<tr>
<th>A.C. Meters</th>
<th>Excitation Supply</th>
<th>D.C. Meters</th>
</tr>
</thead>
<tbody>
<tr>
<td>A V A</td>
<td>V A</td>
<td>V A A A</td>
</tr>
<tr>
<td></td>
<td>Pri</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sec</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 - 150 V D.C 1 A</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Main A. C.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A B C N</td>
</tr>
<tr>
<td></td>
<td>208/120 VAC</td>
</tr>
</tbody>
</table>

|                     |                      |
|                     | A B C N G            |
|                     | 0 - 240 / 140 V A C  |

|                      |                      |
|                      | Pri Sec              |
|                      |                      |
|                      | 0 - 150 VDC 1A       |

|                      |                      |
|                      | Pri Sec              |
|                      |                      |
|                      | 0 - 125 VDC 5A       |
208 / 120 VAC Power Supply
Balanced 3 φ Voltages

0 - 240 / 140 VAC Power Supply
Balanced 3 φ Voltages

The dotted line indicates that the three voltage taps move simultaneously. (They are "ganged together".)
D. C. Power Supplies

Excitation Power Supply
Each relay has the following connections brought out to the banana jacks on the top:

The Relay experimenter is laid out as follows:
AC Power Review

If we wish to transfer power from a sinusoidal source to a complex load, we use the circuit shown in Fig. 1.1 below. Note that the load impedance \( Z \angle \theta \) is a complex number, i.e. the load has both a resistive part and a reactive part. In other words we could write \( Z \angle \theta = R + jX \)

![Diagram showing sinusoidal power transfer with load impedance labeled.](image)

**Fig. 1.1 Sinusoidal Power Transfer**

if \( v(t) = V_p \cos(\omega t) \) then we have the phasor diagram shown in Fig. 1.2 below.

![Diagram showing phasor diagram with load impedance labeled.](image)

**Fig. 1.2 Phasor Diagram**

where

\[
\tilde{I}_p = \frac{\tilde{V}_p}{\tilde{Z}} = \frac{V_p}{Z} \angle -\theta \quad i(t) = \frac{V_p}{Z} \cos(\omega t - \theta)
\]

Now the question is: "If we know the peak values of the voltage and current, and the load impedance, how much average power is delivered to the load?" Since we may write

\[
p(t) = v(t) \cdot i(t) = V_p \cos(\omega t) \cdot I_p \cos(\omega t - \theta)
\]
and since $\cos(a) \cdot \cos(b) = \frac{1}{2} \cdot (\cos(a+b)+\cos(a-b))$ we have

$$p(t) = \frac{V_p I_p}{2} \cos(\theta + \cos(2\omega t - \theta))$$

For example if $\tilde{V}$ and $\tilde{I}$ are as shown below in Fig. 1.3 below,

![Fig. 1.3 Voltage and Current Time Functions](image)

then $p(t)$ is as shown below in fig. 1.4

![Fig. 1.4  Power as a function of Time](image)
There are two notable features of fig. 1.4: the frequency of the power as a function of time is twice the frequency of either the voltage or the current, and the power is sometimes positive and sometimes negative. The significance of the sign of the power is that when the power is positive energy is being transferred from the source to the load, and when the power is negative energy is being transferred from the load to the source. So there are two components of the power: the power that flows unidirectionaly from the source to the load, and the power that is circulating back and forth between the source and the load. To get expressions for these components, we apply

$$\cos(a \pm b) = \cos(a) \cdot \cos(b) \mp \sin(a) \cdot \sin(b)$$

to

$$p(t) = \frac{V_p I_p}{2} (\cos \theta + \cos(2\omega t - \theta))$$

from which we obtain

$$p(t) = V_p I_p \cdot (\cos(\theta) + \cos(\theta) \cdot \cos(2\omega t) + \sin(\theta) \cdot \sin(2\omega t))$$

which can be rearranged to give

$$p(t) = \frac{V_p I_p}{2} \cos \theta \,(1 + \cos 2\omega t) + \frac{V_p I_p}{2} \sin \theta \sin(2\omega t)$$

If we define

$$P = \frac{V_p I_p}{2} \cos(\theta) \quad \text{and} \quad Q = \frac{V_p I_p}{2} \sin(\theta)$$

Then \( p(t) = P \,(1 + \cos(2\omega t)) + Q \sin(2\omega t) \) and

$$P_{AVE} = \frac{1}{T} \int_0^T p(t) \, dt = \frac{1}{T} \int_0^T P \,(1 + \cos(2\omega t)) + Q \sin(2\omega t) \, dt = \frac{1}{T} \,(P)(T) = P$$

P is called the real power (with units of Watts) and represents the AVERAGE power transferred by the source to the load. (There is no such thing as RMS power.) Q is called the reactive power (with units VAR) and represents the amplitude of the power that oscillates back and forth between the source and the load.

We define the complex power \( \bar{S} = P + jQ = S \angle \theta \) and the power factor.
\[ P.F. = \cos(\theta) \quad \text{where} \]
\[ S = \sqrt{P^2 + Q^2} = \frac{V_p \cdot I_p}{2} \quad \theta = \tan^{-1} \frac{Q}{P} \]

Lagging power factor \(\Rightarrow\) Inductive Load, \(\theta > 0\), current lags voltage.

Leading power factor \(\Rightarrow\) Capacitive Load, \(\theta < 0\), current leads voltage.

Power factor =1 (unity power factor) \(\Rightarrow\) Resistive load \(\theta = 0\), current and voltage in phase.

\[ \tilde{Z} = \frac{V_p}{I_p} \angle \theta \]

![Power Triangle Diagram](image)

Fig. 1.5 Power Triangle

The "Power Triangle" in Fig. 1.5 above, is a convenient way to remember the vector relationship among \(P\), \(Q\), \(S\), and \(\theta\), as well as their units. If we express magnitudes in RMS then

\[ v(t) = V_p \cos(\omega t) \rightarrow V \angle 0 \quad \text{where} \quad V = \frac{V_p}{\sqrt{2}} \]

\[ i(t) = I_p \cos(\omega t - \theta) \rightarrow I \angle -\theta \quad \text{where} \quad I = \frac{I_p}{\sqrt{2}} \]

Then \( P = V I \cos(\theta) \quad Q = V I \sin(\theta) \quad \text{and} \quad S = V I \)

The fundamental power formula for AC power is \( \tilde{S} = \tilde{V} \tilde{I}^* \) where the superscript * denotes complex conjugation.

If \( \tilde{Z} = Z \angle \theta = R + JX \), then since \( \tilde{V} = \tilde{Z} \tilde{I} \)
we have \( P = I^2 R \quad Q = I^2 X \)

or

\[
P = \frac{V^2 R}{R^2 + X^2} \quad Q = \frac{V^2 X}{R^2 + X^2}
\]

If we have \( Z \angle \Theta \), we may have

\[
\frac{R_S}{JX_S}
\]

Where \( Z \angle \Theta = R_s + JX_s \) by polar to rectangular conversion of \( Z \angle \Theta \).

That is, the impedance \( Z \angle \Theta \) may represent a resistance in series with a reactance, in which case the resistance has value \( R_s \) and the reactance has value \( X_s \). But that is not the only arrangement of two devices (one resistive and one reactive) whose impedance may be written \( Z \angle \Theta \).

Consider the parallel combination of a resistance and a reactance whose parallel equivalent impedance is \( Z \angle \Theta \). The question is "What combination of resistance \( R_p \) and \( X_p \) have the same impedance \( Z \angle \Theta \) as \( R_s \) and \( X_s \)?"

So we have

\[
Z \angle \Theta = R_p + JX_p
\]

where \( Z \angle \Theta = R_s + jX_s \)

and

\[
R_p = \frac{X_s^2}{R_s} + R_s \quad X_p = \frac{R_s^2}{X_s} + X_s
\]
In addition to adding capacitance across loads to improve their power factor, (see O’Malley Ch. 15) it is sometimes necessary to add capacitance across a resistive load (or a load whose power factor has been adjusted to 1.0 by the methods of O’Malley Ch.15) to improve the voltage profile. Consider the circuit of Fig. 1.6 below.

Fig. 1.6 Resistive Load Fed By Reactive Line

Notice that $\tilde{V}_{Load} < \tilde{V}_s$ due to the voltage drop across the series resistance and reactance of the transmission line. Depending on the currents involved and the length of the transmission line, the voltage drop can become significant. One way to compensate for this is to add a capacitance across the load as shown in Fig. 1.6.

Fig. 1.7 Capacitance to Improve Voltage Profile
We wish to select C so that $|\vec{V}_{Load}| = |\vec{V}_s|$. Since we have, by voltage division:

$$|\vec{V}_{Load}| = \frac{R_L}{j\omega C} \cdot \frac{R_L + \frac{1}{j\omega C}}{\frac{R_L}{j\omega C} + R_{line} + j\omega L_{line}}$$

we may (clearing fractions) write

$$1 = \frac{R_L}{j\omega C} \cdot \frac{\frac{R_L}{j\omega C} + (R_{line} + j\omega L_{line}) \left( R_L + \frac{1}{j\omega C} \right)}{R_L + (R_{line} + j\omega L_{line}) (j\omega C R_L + 1)}$$

or (clearing fractions again)

$$1 = \frac{R_L}{R_L + (R_{line} + j\omega L_{line}) (j\omega C R_L + 1)}$$

So

$$R_L = \left| R_L + R_{line} - \omega^2 C R_L L_{line} + j(\omega L_{line} + \omega C R_L R_{line}) \right|$$

and

$$R_L^2 = \left( R_L + R_{line} - \omega^2 C R_L L_{line} \right)^2 + \left( \omega L_{line} + \omega C R_L R_{line} \right)^2$$

which can be solved by the quadratic formula for C, once numeric values have been substituted for the resistances, the line inductance, and the angular frequency. For example if

$$R_L = 50 \quad R_{line} = 2 \quad L_{line} = 0.01 \quad f = 60$$

then we have

$$50^2 = (50 + 2 - 377^2 C 50 (0.01))^2 + 377^2 (0.01 + C(50)(2))^2$$

or

$$6.4713 \cdot 10^9 C^2 - 7.1065 \cdot 10^6 C + 218.213 = 0$$

Which is easily solved for $C = 31.62 \mu F$, $C = 1066.7 \mu F$. We choose the smaller of the two values, since this produces the smaller capacitor current, and also requires the smaller initial investment.
Experiment 1.  A.C. Power Review (2 weeks)

1. With all breakers open, construct the circuit shown below.

2. Close the main station breaker, Set the scope to trigger on CH 1 (DC) and set the CH 1 and CH 2 inputs to DC. Press the Autoset button and adjust the CH1 and CH2 vert position controls to center both traces. Adjust TRIG level (or adjust the HORIZ position) so that the voltage sine wave crosses through zero (with a positive slope) at the leftmost graticule line.

3. Display $\tilde{V}$ and $\tilde{I}$ together and record them by using the send to USB button, and also record $V$ and $I$. Explain the relationship between the meter readings and the scope readings.

You will note that the current transducer signal is very small and quite noisy. The current transducer used in this lab has a sensitivity of 5V per 20 Amperes or 250 mV per Ampere, so the expected current

$$I = \frac{\sqrt{2} \cdot 120}{500} = 0.3394 A$$
produces a peak voltage of 84.9 mV which is small, noisy and inconvenient for measuring the amplitude of the current waveform.

To combat these problems we must consider how the current transducer works. The current transducer behaves as a transformer whose primary is a single turn (the conductor whose current we are trying to measure), and whose secondary is the coil inside the current transducer.

Since the current probe on the left (in the figure above) has one turn in the primary, the output sensitivity is 1250 mV per Ampere. But since the current probe on the right has two turns in the primary, the sensitivity is 500 mV per Ampere. (We are coupling twice as much flux to the probe.) If we wrap four turns around the transducer, we produce a sensitivity of 1V per Ampere.

3a. Open the main station breaker (to de-energize the 208/120 volt supply), wrap 4 turns of the conductor connecting the two neutrals together around the current transducer (these turns must be neat and tight), close the main station breaker (which re-energizes the 208/120 volt supply), and display and record $\tilde{V}$ and $\tilde{I}$ together. (Keep in mind that the current probe sensitivity is now 1 V per Ampere.)
4. Open the main breaker and connect a reactance load in parallel with load resistor. Turn the reactance knob to maximum LAG, close the main breaker and display and record $V$ and $I$ together.

5. Rotate the knob clockwise to full lead observing the phase relationship between $V$ and $I$ as you do. When the knob is at full lead display, and record $V$ and $I$ together.

6. Open the breaker and construct the following circuit. (We now present an alternative method of finding $\theta$ that does not require an oscilloscope.)

7. Set the reactance control to zero, close the breaker and measure $P_1$, $V_1$, and $I_1$ by pressing the appropriate buttons on the power meter front panel. Use the power reading $P$ along with $V$ and $I$ to calculate $\theta$ as

$$\theta = \cos^{-1} \left( -\frac{P_1}{V_1 \cdot I_1} \right)$$

(For completeness, and for later corroboration, use the appropriate buttons to measure $\text{Var}$ [the reactive power] and the PF.)

8. Set the reactance control to max LAG and measure $P_2$, $V_2$, and $I_2$.

$$\theta = \cos^{-1} \left( -\frac{P_2}{V_2 \cdot I_2} \right)$$

9. Repeat 8. for max lead.

10. Construct the circuit shown below. (Measure the resistance and reactance of the two 8 mH coils before placing them in the circuit.)
11. Energize the supply, and measure \( V_S \) and \( V_L \) (using the same Fluke for both measurements), and the line current \( I \). (One can measure the RMS equivalent of the line current by measuring the current transducer output with a Fluke. DMM)

12. De-energize the supply, and connect all three reactances of the RLC - 100 in parallel. Set the lead/lag knob of each to its mid position, and use the resulting reactor (it will be a capacitor when we rotate the lead/lag knob of any of the reactances to lead) to construct the circuit shown below.
13. Energize the supply, rotate the reactor knobs clockwise (toward LEAD) until

$$|\bar{V}_L| = |\bar{V}_S|$$

and measure the reactor current and the line current.

14. Compare your result in 13. with the theoretical prediction.

15. Calculate the load impedance $Z_1$ at angle $\theta_1$ from the result in 4. and find the value of the inductor in parallel with the 500 $\Omega$ resistor.

16. Calculate the load impedance $Z_2$ at angle $\theta_2$ from the result in 5. and find the value of the capacitor in parallel with the 500 $\Omega$ resistor.

17. Use the result from 8. and 9. to recompute $\tilde{Z}_1$ and $\tilde{Z}_2$ and compare to the result of 15. and 16.

18. Use the result of 13. To calculate the value of the capacitance you have added across the load, and compare to the theoretical prediction.
Single-phase Supplemental Power Problems

#1. A single phase 220 V load draws 10 kW and 10 kVAR. If the load is fed by a transmission line whose Resistance is 0.1Ω and whose Reactance is 0.4Ω, find the voltage at the sending end of the line.

#2. A single phase 220 V load draws 100 kVA at pf = 0.8 lagging. If the load is fed by a transmission line whose Resistance is 0.02Ω and whose Reactance is 0.1Ω, find the voltage at the sending end of the line.

#3. A single phase 13200 V load draws 6.0 MW at pf = 0.9 lagging. If the load is fed by a transmission line whose Resistance is 0.1Ω and whose Reactance is 0.5Ω, find the voltage at the sending end of the line.
Balanced Three-Phase Systems

If we connect three sinusoidal voltage sources as shown in Fig. 2-1, we form a Y connected three-phase system.

![Diagram of a Y-connected three-phase system with voltage sources labeled as follows: $V_{an}$, $V_{bn}$, and $V_{cn}$ at the nodes a, b, and c, respectively. The diagram also shows the phase voltages $V_{ab}$, $V_{bc}$, and $V_{ca}$ at the nodes a, b, and c, respectively.]

Figure 2-1 Three-Phase System

If we restrict the amplitudes and phases of the sources such that we have $\tilde{V}_{an} + \tilde{V}_{bn} + \tilde{V}_{cn} = 0$ and $|V_{an}| = |V_{bn}| = |V_{cn}|$ then the result is called a balanced three-phase system. Three vectors that satisfy the above phasor requirement are shown in Fig. 2-2.
It is very important to understand what the above vector diagram means. The diagram represents three sinusoids that have the same magnitude but differ in phase from one another by 120° as shown in Fig. 2-3.

Now both the vector diagram and the time waveform show \( \tilde{V}_{an} \) with a phase of zero - in other words crossing through zero at time \( t = 0 \), but this is not required for a system to be a balanced three phase system. All that is required is that the magnitudes be equal and that the phases differ by 120°. For an example of a balanced three phase system where \( \tilde{V}_{an} \) does not have zero phase, examine fig. 2-4 and fig. 2-5 where we have shown a balanced three-phase system wherein the phase of \( \tilde{V}_{an} \) is 45°.
Since there are many instances where the neutral connection is not accessible (or does not even exist), three-phase systems are not described in terms of their phase voltages

\[ (\tilde{V}_{an}, \tilde{V}_{bn}, \tilde{V}_{cn}) \]

by power engineers. Rather, three-phase systems are described in terms of their line voltages

\[ (\tilde{V}_{ab}, \tilde{V}_{bc}, \tilde{V}_{ca}) \]

where

\[ \tilde{V}_{ab} = \tilde{V}_a - \tilde{V}_b \]
\[ \tilde{V}_{bc} = \tilde{V}_b - \tilde{V}_c \]
\[ \tilde{V}_{ca} = \tilde{V}_c - \tilde{V}_a \]
Using the above definition, a little complex algebra gives

\[ \hat{V}_{ab} = \hat{V}_a \cdot \sqrt{3} \angle 30^\circ \quad \hat{V}_{bc} = \hat{V}_b \cdot \sqrt{3} \angle 30^\circ \quad \hat{V}_{ca} = \hat{V}_c \cdot \sqrt{3} \angle 30^\circ \]

The line voltages and the phase voltages are shown as a function of time in fig. 2-6.

![Graph showing line voltages as a function of time](image)

**Figure 2-6** Line Voltages as a Function of Time

Since the time-function diagram can become quite cluttered and therefore difficult to read, most engineers prefer to use the phasor diagram shown in Fig. 2-7. Figures 2-6 and 2-7 contain exactly the same information, but Fig. 2-7 shows the information in a more convenient format.
Figure 2-7  Line Voltage Phasors

Now if we load a balanced three phase source with a balanced three-phase, Y connected load as
shown in Fig 2-8, then balanced three-phase currents flow as shown in Fig. 2-9.

Figure 2-8  Three Phase System with Y Connected Load

The balanced three-phase line currents \((\tilde{I}_a, \tilde{I}_b, \tilde{I}_c)\) are related to the phase voltages and the line voltages as shown in Fig. 2-9 and 2-10. (\(\theta \) has been set arbitrarily to 20° and \(|I|\) has been set to \(0.8 \cdot |V|\) for purposes of illustration in Figures 2-9 and 2-10.)
Figure 2-9  Three Phase System with Y Connected Load

Figure 2-10  Line Current Phasors for Y connected Load
Now by examination of Figures 2-11 and 2-12, we can see that the sum of the three line currents

\[ \tilde{I}_a + \tilde{I}_b + \tilde{I}_c \]

is zero at every time \( t \).

Figure 2-11 Three-phase Currents

Figure 2-12 Line Current Phasors
If we write Kirchhoff's Current Law at the neutral terminal of Fig. 2-8 (it does not matter whether we write KCL at the node marked n or at the node marked N) we have that

$$\vec{I}_a + \vec{I}_b + \vec{I}_c + \vec{I}_n = 0$$

But since the line currents form a balanced three-phase set, we have by definition

$$\vec{I}_a + \vec{I}_b + \vec{I}_c = 0$$

Taken together these equations constrain the neutral current ($\vec{I}_n$) to be zero. What this means is that $\vec{V}_n = \vec{V}_N$, and that the three loop equations (one for phase a, one for phase b, and one for phase c) are completely decoupled. That is to say, the loop equation for phase a depends only on $\vec{V}_a$ and $\vec{I}_a$, not on $\vec{V}_bn$, $\vec{V}_cn$, $\vec{I}_b$, or $\vec{I}_c$. This means that the equations can be solved independently instead of simultaneously. Therefore whether the actual circuit contains a neutral connection or not, a neutral connection is assumed and the "per phase diagram" is constructed as shown in Fig. 2-13.

![Diagram](image)

Figure 2-13

$\vec{I}_a$ is found easily as $\vec{I}_a = \frac{\vec{V}_a}{\vec{Z}}$ and $\vec{I}_b$ and $\vec{I}_c$ are found from $\vec{I}_b = \vec{I}_a \cdot 1 \angle -120^\circ$ and $\vec{I}_c = \vec{I}_a \cdot 1 \angle 120^\circ$

The Y connected load is not the only possible balanced load connection for a three-phase system. Another balanced connection, called the $\Delta$ connection is possible, and is shown in Fig. 2-14.
Figure 2-14  Three Phase System with Δ Connected Load

Note that there is no neutral connection at the load so the voltage $\tilde{V}_{AN}$ has no physical significance. This is one of the reasons why power engineers always discuss three-phase systems in terms of the line voltages, which always have physical significance. Since we have found the per phase analysis to be extremely useful, we are led to ask if there might be a way to extend the method to Δ connected systems. What we require is a Y connected load that is in some sense equivalent to the original Δ connected load. Such an equivalence is provided by the Δ Y transformation as illustrated in Fig. 2-15.
Figure 2-15  The Delta - Y Transformation

Our plan for solving circuits like the one shown in Fig. 2-14, is to do a Δ Y transformation of the balanced Δ connected load then do a per phase analysis to find the line currents $\tilde{I}_{aA}, \tilde{I}_{bB}$, and $\tilde{I}_{cC}$. Notice, however, that the line currents (which form a balanced three phase system as shown in Figures 2-11 and 2-12) do not flow in the individual legs of a Δ connected load. The currents in the individual loads are $\tilde{I}_{AB}, \tilde{I}_{BC}$, and $\tilde{I}_{AB}$ where

$$\tilde{I}_{AB} = \frac{1}{\sqrt{3}} \angle 30^\circ \cdot \tilde{I}_{aA} \quad \tilde{I}_{BC} = \frac{1}{\sqrt{3}} \angle 30^\circ \cdot \tilde{I}_{bB} \quad \tilde{I}_{CA} = \frac{1}{\sqrt{3}} \angle 30^\circ \cdot \tilde{I}_{cC}$$
These relationships are shown in the phasor diagram of Fig. 2-16.

Figure 2-16  Line Current and Load Current Phasors for Δ Connected Load

The voltages $\tilde{V}_{AN}$, $\tilde{V}_{BN}$, and $\tilde{V}_{CN}$ are the voltages across the appropriate leg of the Y transformed circuit of the original Δ connected load.

**Three Phase Power**

The total power drawn by a balanced, Y-connected load is

$$P_T = 3 \cdot I_{Line} \cdot V_{Phase} \cdot \cos(\theta) = 3 \cdot I_{Line} \cdot \frac{V_{Line}}{\sqrt{3}} \cdot \cos(\theta)$$

2-12
and the total power drawn by a balanced, Delta-connected load is

$$P_T = 3 \cdot V_{Line} \cdot I_{Phase} \cdot \cos(\theta) = 3 \cdot \frac{I_{Line}}{\sqrt{3}} \cdot V_{Line} \cdot \cos(\theta)$$

The total power drawn by a three phase load can be written as

$$P_T = \sqrt{3} \cdot I_{Line} \cdot V_{Line} \cdot \cos(\theta)$$

regardless of whether the load is Y or Delta connected.

The total power drawn by a balanced three-phase load can be measured with a single wattmeter connected as shown below in Fig. 2-17 but the wattmeter reading must be multiplied by 3, and it is necessary that the neutral be available (which is not always the case).

A second method, using two wattmeters, has the advantage that no multiplications are necessary to find the total power: it is only necessary to add the readings of the two wattmeters together to obtain the total power. A second advantage of the two wattmeter method (shown in Fig. 2-18 below) is that it works whether or not the neutral is available, and it works on unbalanced loads as well as on balanced loads.
Now the wattmeter 1 reading is $|V_{ab}| |I_a| \cos(30^\circ + \theta)$ (see Fig. 2-10), and the wattmeter 2 reading is $|V_{cb}| |I_c| \cos(30^\circ - \theta)$ (remember $\vec{V}_{cb} = -\vec{V}_{bc}$). The sum of the two readings is

$$V_L \cdot I_L \cdot (\cos(30^\circ + \theta) + \cos(30^\circ - \theta))$$

Since $\cos(30^\circ + \theta) = \cos(30^\circ) \cdot \cos(\theta) - \sin(30^\circ) \cdot \sin(\theta)$, and

$\cos(30^\circ - \theta) = \cos(30^\circ) \cdot \cos(\theta) + \sin(30^\circ) \cdot \sin(\theta)$, we have

$$V_L I_L (\cos(30^\circ + \theta) + \cos(30^\circ - \theta)) = V_L I_L (2 \cos(30^\circ) \cos(\theta)) = V_L I_L \sqrt{3} \cos(\theta)$$

So the algebraic sum (it is necessary to include the sign, since one or both of the wattmeter readings may be negative) of the wattmeter readings is the total three-phase power. (This analysis, of course depends on the fact that the system is balanced. A derivation for the unbalanced case is beyond the scope of this course.)
The power factor can be calculated directly from the power meter readings as

\[
P.F. = \frac{1}{\sqrt{1 + 3 \left( \frac{1 - \frac{P_1}{P_2}}{1 + \frac{P_1}{P_2}} \right)^2}}
\]

The reactive power \( Q \) of a balanced three-phase load can be measured with a four terminal wattmeter by placing the current coil in series with line aA and placing \( \vec{V}_{be} \) across the potential coil. The wattmeter reads \( V_L I_L \cos(90^\circ - \theta) = \sqrt{3} \ V_{ph} I_{ph} \sin(\theta) = \frac{Q}{\sqrt{3}} \) (See Fig. 2-10)

This arrangement is not possible for the (essentially) three terminal wattmeters in the lab without the use of an isolation transformer.
Experiment 2. Three-Phase Systems (3 weeks)

1. Construct the circuit shown below.

![Circuit Diagram](image)

0-240/140 VAC

Close the supply breaker, adjust the phase voltage to 120 $V_{\text{rms}}$, get a stable trace as shown below and record $\tilde{V}_{an}$. (Trigger from ch1; ch2 and ch3 should be off for this.)

(Use send to usb to record $\tilde{V}_{an}$.)
2. Turn on the ch2 and ch3 displays and display \( \tilde{V}_{an}, \tilde{V}_{bn}, \tilde{V}_{cn} \) together.

3. Use the math function to add a display of (ch1 - ch2). Display and record (use send to usb to record) \( \tilde{V}_{an}, \tilde{V}_{bn}, \tilde{V}_{cn} \) and \( \tilde{V}_{ab} \) together.

4. Use ch3 for phase b and ch4 for phase c (keep ch1 on phase a, and continue to trigger form phase a), and display and record \( \tilde{V}_{an}, \tilde{V}_{bn}, \tilde{V}_{cn} \) and \( \tilde{V}_{bc} \) together.

5. Use ch1 for phase a, ch2 for phase c and ch3 for phase b. and display and record \( \tilde{V}_{an}, \tilde{V}_{bn}, \tilde{V}_{cn} \) and \( \tilde{V}_{ca} \) together.

6. Construct the circuit shown below with the supply set to 208 \( V_{RMS} \) (line-to-line). While only one current transducer is shown in the aA line, you can save some time by placing current transducers in line bB and line cC as well. Then you can simply move the scope connections to make the measurements indicated in step 7.
7. a) Display and record $\tilde{V}_{an}$ and $\tilde{I}_{aA}$ together, $\tilde{V}_{bn}$ and $\tilde{I}_{bB}$ together, $\tilde{V}_{cn}$ and $\tilde{I}_{cC}$ together, and record $I_n$.
b) Remove the nN connection and repeat a).

8. Add 500 $\Omega$ in parallel with CN and repeat 7.

9. Remove the 500 $\Omega$ parallel resistance from across CN, change the load resistance to 500 $\Omega$ per phase, add the 3 phase reactance load in parallel (set to maximum lag) with the resistive load and keep the supply at 208 V line to line (120 V line to neutral) as shown below.
10. Measure and record $\vec{V}_{an}$ and $\vec{I}_{aA}$ together, $\vec{V}_{bn}$ and $\vec{I}_{bB}$ together, and $\vec{V}_{cn}$ and $\vec{I}_{cC}$ together.

11. While observing $\vec{V}_{an}$ and $\vec{I}_{aA}$ together, turn all 3 reactance knobs to full Lead and measure and record $\vec{V}_{an}$ and $\vec{I}_{aA}$ together, $\vec{V}_{bn}$ and $\vec{I}_{bB}$ together, and $\vec{V}_{cn}$ and $\vec{I}_{cC}$ together.

(Ignore the distortion you see when the knob is at full Lead - this distortion is a result of the way the RLC - 100 produces reactive loads.)
12. Construct the circuit shown below. (Trigger on Van, and set the supply to 208 V line to line (120 V line to neutral).)

The best way to do this is shown below.
13. Measure and record $\tilde{V}_{an}$ and $\tilde{I}_{aA}$ together, $\tilde{V}_{an}$ and $\tilde{I}_{AB}$ together, $\tilde{V}_{bn}$ and $\tilde{I}_{bB}$ together, $\tilde{V}_{bn}$ and $\tilde{I}_{BC}$ together, $\tilde{V}_{cn}$ and $\tilde{I}_{cC}$ together, and $\tilde{V}_{cn}$ and $\tilde{I}_{CA}$ together.

14. Add 500Ω in parallel with BC and repeat (13).

15. Remove the 500Ω from across BC, remove the connections that put the resistors in a delta configuration, add a reactance load (Max Lag) in parallel with each leg of the load, and place the three composite loads in a Delta configuration. (If you simply add the reactances across the Delta connected resistors, you may be unable to isolate the phase currents, depending on how you put the resistors in Delta.)

16. Measure and record $\tilde{V}_{an}$ and $\tilde{I}_{aA}$ together and $\tilde{V}_{an}$ and $\tilde{I}_{AB}$ together, $\tilde{V}_{bn}$ and $\tilde{I}_{bB}$ together and $\tilde{V}_{bn}$ and $\tilde{I}_{BC}$ together, and $\tilde{V}_{cn}$ and $\tilde{I}_{cC}$ together and $\tilde{V}_{cn}$ and $\tilde{I}_{CA}$ together.

17. While observing $\tilde{V}_{an}$ and $\tilde{I}_{aA}$ together, turn all 3 reactance knobs to full Lead and measure and record $\tilde{V}_{an}$ and $\tilde{I}_{aA}$ together and $\tilde{V}_{an}$ and $\tilde{I}_{AB}$ together, $\tilde{V}_{bn}$ and $\tilde{I}_{bB}$ together and $\tilde{V}_{bn}$ and $\tilde{I}_{BC}$ together, and $\tilde{V}_{cn}$ and $\tilde{I}_{cC}$ together and $\tilde{V}_{cn}$ and $\tilde{I}_{CA}$ together.

18. Construct the circuit shown below.
Wattmeter 1 reads $-\tilde{I}_a \cdot \tilde{V}_{ba} = \tilde{I}_a \cdot \tilde{V}_{ab}$ and

wattmeter 2 reads $-\tilde{I}_c \cdot \tilde{V}_{bc} = \tilde{I}_c \cdot \tilde{V}_{cb}$ so the readings add to produce the total three-phase power as in Fig. 2-18.

19. Set R to 1000 $\Omega$ per phase, set the reactance knobs to their midposition, the supply to 208 V Line to Line (120 V Line to Neutral), and measure $P_1$, $P_2$, and $P_T$.

20. Rotate the reactance knobs to full lag and measure $P_1$, $P_2$, and $P_T$.

21. Rotate the reactance knobs to full lead and measure $P_1$, $P_2$, and $P_T$. 

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22. Using the relevant circuit theory, calculate the values for the magnitude and phase of each quantity you have measured and calculate the % Error.

23. Produce a phasor diagram from the data of (3)

24. Produce a phasor diagram from the data of (4)

26. Produce a phasor diagram from the data of (6)

27. Produce a phasor diagram from the data of (10)

28. Produce a phasor diagram from the data of (11)

29. Produce a phasor diagram from the data of (13)

30. Produce a phasor diagram from the data of (14)

31. Produce a phasor diagram from the data of (16)

32. Produce a phasor diagram from the data of (17)

33. Calculate the power factor for (19), (20), and (21).
Three-phase Supplemental Power Problem

Two 13.2 kV, three-phase loads are connected in parallel and fed by a transmission line whose impedance is $2 + j4$ ohms per phase. Load 1 is $Y$ connected, draws 160 kW at PF 0.8 lagging, load 2 is delta connected and draws 130 kVAR at PF 0.9 lagging. The transmission line is fed by a $Y$ connected three-phase generator whose internal impedance is $0.6 + j1$ ohms per phase. Find a) the voltage at the terminals of the generator, b) the value of capacitance that would have to be connected across the lines at the load in a delta configuration to produce a unity load power factor, c) the impedance of each leg of each load.
One of the first commercial uses to which the magnetomotive force was put, was the 
electromagnetic telegraph. The telegraph is basically an electromagnet that attracts a small piece 
of iron (the armature) when it is on, and releases the armature when it is off. When the magnet is 
off a small spring returns the armature to its original position. (See Fig. 3-1 below.)

![Fig. 3-1 Telegraph Sounder](image)

The device is used to send messages by placing the battery and switch in one location, and the 
electromagnet (the sounder) in another location. An observer at the remote location can then tell 
whether the switch is open or closed by observing the sounder's armature. If the sender 
manipulates the switch according to some preset code (Morse code usually), messages can be sent 
over relatively long distances. However there is a limit to the distance over which a message can 
be sent in this manner, since the voltage drop across a long wire (the wire has some finite 
resistance) becomes so great that there is insufficient power left to move the sounder's armature. 
Telegraph engineers soon realized that they could attach switch contacts to the sounder's 
armature that would mimic the operation of the sender's switch. The position of the sender's 
switch could then be "relayed" down the line of a series of such devices, and telegraph messages 
could be sent over any distance.

![Fig. 3-2 Relay](image)
The use of the relay for sending telegraph messages over long distances is shown in Fig. 3-3 below.

![Relay Station diagram]

Fig. 3-3 Telegraph Relay Station

The relays shown above are called single pole (only one set of contacts), single throw (the contacts are either open or closed) relays. Single throw, normally open contacts are often referred to as form A contacts. If a second contact surface is added as shown in Fig. 3-4, below, we form a single pole double throw relay. Double throw contacts are often referred to as form C contacts.

The term "normally" when used to describe a set of relay contacts, refers to the case when there is no current flowing in the coil that controls that contact. So, a normally open contact is open if there is no current flowing in the coil that controls it, but closed when current is flowing in the coil that controls it. Similarly, a normally closed contact is closed when there is no current in the coil that controls it, and open when current is flowing in the coil that controls it.
Relays are depicted schematically as shown in Fig. 3-5 below.

Relay coils are given names (CR1, CR2, ... or K1, K2, ...) and contacts are referenced by the name of the coil which actuates them. Although the relays we have seen so far have only one set of contacts each, relays with 2 - 6 sets of contacts actuated by a single coil are not uncommon. Remember that you cannot tell whether a contact is open or closed by looking at the contact symbol: normally open contacts can be either open or closed depending on whether or not current is flowing in the coil that controls the normally open contacts. Similarly normally closed contacts
can be either closed or open depending on whether or not current is flowing in the coil that controls the normally closed contacts.

One common use of relays is to allow the use of low voltages switched by push-button switches to control the operation of A.C. powered devices. Consider the circuit shown in Fig. 3-6 below.

![Control Relay Circuit Diagram](image)

**Fig. 3-6**  
Control Relay

When Push Button 1 (PB-1) is depressed (closing its contacts) CR-1's coil is energized, applying 110 V A.C. to the lamp. Simultaneously, CR-1's other contacts close around PB-1 so that when the button is released and its contacts open, the coil of CR-1 is still supplied through CR-1's own contacts and the lamp stays on. If not for the contacts around PB-1, the lamp would extinguish as soon as the button were released. This technique is referred to as sealing the push button, and the contacts around PB-1 are referred to as the sealing contacts. The lamp is turned off by pressing PB-2 (thus opening its contacts) which deenergizes CR-1's coil, which releases the sealing contacts so that when PB-2 is released the lamp does not come on again until PB-1 is pressed again. PB-1 is the "on" switch and PB-2 is the "off" switch.

It is important to remember that since the contacts are completely isolated from the coil, the contact electrical ratings may be quite different from the coil electrical ratings. For instance, in the example above the coil may be rated at 24 V, 500 mA D.C., while the contacts might be rated at 110 V, 5 Amps A.C. It is also possible to build relays with coils rated for A.C. voltages, but their use is limited by eddy current losses in the coil core laminations. Relays that have A.C. coils
also tend to hum audibly when energized.

If one slowly increases the voltage to a D.C. relay's coil, the armature does not slowly move from its de-energized state to its energized state. Instead the armature remains motionless until the coil voltage reaches a critical level called the pull in voltage, at which point the armature snaps closed. When the coil voltage is decreased slowly the armature remains motionless until the voltage reaches a second critical voltage, called the drop out voltage, at which point the armature snaps back to its original position. The drop out voltage is significantly smaller than the pull in voltage. To understand this somewhat counter-intuitive phenomenon, we recall that the force available to move objects is directly proportional to \( \vec{B} \), the magnetic flux density, which in turn is in inversely proportional to the length of the air gap between the armature and the coil core. When the coil voltage reaches the pull in voltage, the armature moves slightly toward the coil core, which reduces the size of the air gap, which increases the value of \( \vec{B} \), which increases the attractive force on the armature, which causes the armature to move closer to the coil core, decreasing the gap etc. This process builds up quickly, snapping the armature shut in a very short time. After the armature is closed the gap is very small indeed, and the coil voltage can be reduced considerably before \( \vec{B} \) drops sufficiently to release the armature. This is why the drop out voltage is always smaller than the pull in voltage.

The effect that varying the coil voltage of a relay linearly up and down would have on the position of the contacts is shown by the plots in Fig. 3-7 below. The fact that the pull-in voltage is different from the drop-out voltage is called hysteresis (this is a different phenomenon from hysteresis in a B H curve), and the fact that the pull-in voltage is not zero is described by saying that the relay has a "dead-zone". The hysteresis and dead zone effects are non-linear effects that can cause problems in automatic control systems that contain them. These effects can cause oscillations called limit cycles that are not predicted by linear control theory.
Fig. 3-7  Contact Position Versus Coil Voltage
Experiment 3. Magnetic Circuits   Relays

1) With ALL breakers open (OFF), construct the following circuit.

2) With the D.C. supply control knob fully CCW, close (turn on) the Main A.C. breaker, then the D.C. supply primary winding and secondary winding breakers.

3) SLOWLY increase the D.C. supply voltage to 12 V, noting the voltage at which the relay armature changes position (V_{pull in}). SLOWLY decrease the D.C. supply to 0 V, noting the voltage at which the relay armature returns to its original position (V_{drop out}). Record V_{pull in} and V_{drop out}.

4) With ALL breakers open, construct the following circuit.

5) With the A.C. supply control knob fully CCW, close the Main A.C. breaker, then the 0 - 240 / 140 V A.C. supply breaker.
6) SLOWLY increase the A.C. supply voltage to 120 V, noting the voltage at which the relay armature changes position (\( V_{\text{pull-in}} \)). SLOWLY decrease the D.C. supply to 0 V, noting the voltage at which the relay armature returns to its original position (\( V_{\text{drop-out}} \)). Record \( V_{\text{pull-in}} \) and \( V_{\text{drop-out}} \).

7) With all breakers open, construct the following circuit. (Use the panel meters not the DMM)

![Circuit Diagram]

0 - 125 V D.C.

8) With the 0 - 125 V D.C. 5A supply control fully CCW, close the main breaker, then the D.C. supply primary winding and secondary winding breakers. With the control fully CCW, close the 0 - 240/140 V A.C. supply breaker, then bring the supply voltage to 50 V. Record the D.C. supply voltage, the A.C. supply voltage, and the states of Lamp #1 and Lamp #2. Set the D. C. supply to 12 V, and record the D.C. supply voltage, the A.C. supply voltage, and the states of the lamps. Depress the button and record the new states of the lamps.
9) With all breakers open, modify the circuit of part 7) and 8) as follows:

\[0 - 125 \text{ V D. C.}\]

10) Repeat 8) for the above circuit.

11) Briefly describe why each lamp is on or off for each part of 8) and 10).

12) With all breakers open, construct the following circuit.
13) With the 0 - 125 V D. C. 5A supply control fully CCW, close the main breaker, then the D. C. supply primary winding and secondary winding breakers. With the control fully CCW close the 0 - 240/140 V A.C. supply breaker, then bring the supply voltage to 50 V. Record the D.C. supply voltage, the A.C. supply voltage, and the state of the Lamp. Set the D. C. supply to 12 V, and record the new state of the Lamp. Depress and release the button PB-1, and record the new state of the Lamp. Depress and release the button PB-2, and record the new state of the Lamp.

14) Describe the operation of the circuit constructed in (12) and explain why it works the way it does.
Magnetic Flux Load-line Problem

This problem has two features which make it attractive for a discussion of numerical techniques available in Maple: the B-H relation is given as a table, requiring a curve-fit before analysis can begin, and the equations describing the problem are tightly coupled, requiring an extension of Newton's method to vector equations.

Curve Fitting

We introduce two approaches to fitting a polynomial to a given set of points: the interpolating polynomial and the least-squares-fit-polynomial. The interpolating polynomial is described first on a simple example for ease of derivation. We must not lose sight of the fact, however, that our goal is to find a polynomial that passes through selected points of the BH curve for a magnetic material.

Interpolating Polynomials

Let us suppose that we are given four points that satisfy an unknown function:

\[(x_1, y_1), (x_2, y_2), (x_3, y_3), \text{ and } (x_4, y_4)\]

We wish to specify a polynomial that passes through the given points: such a polynomial is called an interpolating polynomial. We recall that a line \(y = ax + b\) has two degrees of freedom (\(a\) and \(b\)), and is determined by two points, a parabola \(y = ax^2 + bx + c\) has three degrees of freedom (\(a\), \(b\), and \(c\)) and is determined by three points; in general an \(n^{th}\) order polynomial is determined by \((n+1)\) points. We, therefore, seek a cubic polynomial passing through the given points. A cubic polynomial has the form

\[a \cdot x^3 + b \cdot x^2 + c \cdot x + d\]

This is a third order polynomial so \(n=3\).

Since each of the given points must satisfy the polynomial we must have

\[a \cdot x_1^3 + b \cdot x_1^2 + c \cdot x_1 + d = y_1\]

\[a \cdot x_2^3 + b \cdot x_2^2 + c \cdot x_2 + d = y_2\]

\[a \cdot x_3^3 + b \cdot x_3^2 + c \cdot x_3 + d = y_3\]

\[a \cdot x_4^3 + b \cdot x_4^2 + c \cdot x_4 + d = y_4\]

So we have four equations in the four unknowns \(a\), \(b\), \(c\), and \(d\). (Remember that the points \((x_1, y_1), (x_2, y_2), (x_3, y_3), \text{ and } (x_4, y_4)\) are given.

Rewriting the equations in matrix form we have

\[
\begin{bmatrix}
\Rightarrow
y_1 \\
\Rightarrow
y_2 \\
\Rightarrow
y_3 \\
\Rightarrow
y_4
\end{bmatrix} \Rightarrow \text{coef} = \begin{bmatrix}
a \\
b \\
c \\
d
\end{bmatrix} \Rightarrow \begin{bmatrix}
x_1^3 & x_1^2 & x_1 & 1 \\
x_2^3 & x_2^2 & x_2 & 1 \\
x_3^3 & x_3^2 & x_3 & 1 \\
x_4^3 & x_4^2 & x_4 & 1
\end{bmatrix}
\]
Our polynomial is defined in general as

\[ P = \sum_{i=1}^{n+1} \text{coef}_i x^{n+1-i} \]

For example suppose the four given points are: (0,0) (1,1) (2,-1) (3,3).

To use the built-in curvefitting functions, we first define vectors \( X \) and \( Y \)

\[
X := \begin{bmatrix}
0 \\
1 \\
2 \\
3
\end{bmatrix}
\]

\[
Y := \begin{bmatrix}
0 \\
1 \\
-1 \\
3
\end{bmatrix}
\]

Then we invoke the built-in curve-fitting function as

\[
\text{CurveFitting}[\text{PolynomialInterpolation}](X, Y, x) = \frac{3}{2} x^3 - 6 x^2 + \frac{11}{2} \]

\[
\text{points} := \text{plots}[\text{pointplot}](\{[0, 0], [1, 1], [2, -1], [3, 3]\}) :
\]

\[
\text{poly} := \text{plot}(3, x = -0.5 .. 3.5, -2 .. 5) :
\]

\[
\text{plots}[\text{display}](\{\text{points, poly}\})
\]
The BH Curve for Silicon Sheet Steel

The BH curve of a magnetic material is determined by the manufacturer of the material by subjecting the material to a known $H=N\cdot I$ and measuring the resulting $B$. (This is done with a gauss-meter or a hall-effect device.)
We select a number of evenly spaced points to describe the graph (Ten points is usually sufficient.), thereby defining our data vectors \( x_v \) and \( y_v \). These points are read directly off the given graph.

The \( x \) data vector is

\[
x_v := \text{Vector}(11, [0.0, 100.0, 200.0, 300.0, 400.0, 500.0, 600.0, 700.0, 800.0, 900.0, 1000.0])
\]

By default Maple refuses to display any Matrix or Vector with any dimension larger than ten. The Matrix browser can be invoked by double-clicking in the blue output area of (4). To force Maple to display 15 by 15 matrices we use the interface command as show below.

\[
\text{interface(rtablesize = 15)}
\]
The y data vector is

\[ yv := \text{Vector}(11, [0.0, 0.7, 0.99, 1.125, 1.2, 1.245, 1.275, 1.3, 1.325, 1.34, 1.35]) \]

\[ \begin{bmatrix}
0. \\
0.7 \\
0.99 \\
1.125 \\
1.2 \\
1.245 \\
1.275 \\
1.3 \\
1.325 \\
1.34 \\
1.35 \\
\end{bmatrix} \]

\[ \text{CurveFitting}[\text{PolynomialInterpolation}](xv, yv, H) \]

\[ 3.720234182 \times 10^{-28} H^{10} - 1.825670578 \times 10^{-24} H^9 + 3.844242413 \times 10^{-21} H^8 - 4.506857754 \times 10^{-18} H^7 \\
+ 3.157722956 \times 10^{-15} H^6 - 1.273450786 \times 10^{-12} H^5 + 2.069255224 \times 10^{-10} H^4 + 5.620890933 \times 10^{-8} H^3 \\
- 0.00004013888358 H^2 + 0.01034478191 H \]  

\[ \text{points1} := \text{plots}[\text{pointplot}](xv, yv) : \]

\[ \text{poly1} := \text{plot}((8), H=0..1000) : \]

\[ \text{plots}[\text{display}](\{\text{points1}, \text{poly1}\}) \]
We can see from the above plot that the polynomial contains the required points, but the polynomial is tenth order. We are naturally led to ask if a lower order polynomial could be fit to the data. Of course, the answer is yes and we introduce the following discussion of the least squares fit. Also the interpolating polynomial is only guaranteed to pass through the required points, and might oscillate wildly between the points.

**Least Squares Polynomial Fit**

If we have \( k \) pairs of points that we wish to fit with an \( n^{th} \) order polynomial \((k > n)\), we have \( k \) polynomials of the form:

\[
y_i = a \cdot x_i^3 + b \cdot x_i^2 + c \cdot x_i + d
\]

where we have set \( n = 3 \) for simplicity in the following derivation.

Now if \( n = (k-1) \), then each of the above equations would hold, since the polynomial would actually contain all \( k \) pairs of points. However, since the polynomial only approximates the data, the above equations do not hold. In fact the difference between the left-hand side and the right-hand side of the \( i \)th equation is the error of the polynomial at the \( i \)th data pair. (The red is the polynomial, while the blue represents the given points.)
We seek a polynomial that minimizes the sum of the squares of all the errors. (We must minimize the squares of the errors, since we don't want positive and negative errors canceling each other out, which would make us think we had a better approximation than we actually have.) We must, then, minimize the following error function:

$$F = \sum_{m=1}^{k} \left( y_m - \left( a \cdot x_m^3 + b \cdot x_m^2 + c \cdot x_m + d \right) \right)^2$$

For $F$ to have a minimum we require

$$\frac{\partial}{\partial a} F = \frac{\partial}{\partial b} F = \frac{\partial}{\partial c} F = \frac{\partial}{\partial d} F = 0$$

Carrying out the indicated differentiations gives

$$\sum_{m=1}^{k} y_m \cdot x_m^3 = \sum_{m=1}^{k} a \cdot x_m^6 + \sum_{m=1}^{k} b \cdot x_m^5 + \sum_{m=1}^{k} c \cdot x_m^4 + \sum_{m=1}^{k} d \cdot x_m^3$$

$$\sum_{m=1}^{k} y_m \cdot x_m^2 = \sum_{m=1}^{k} a \cdot x_m^5 + \sum_{m=1}^{k} b \cdot x_m^4 + \sum_{m=1}^{k} c \cdot x_m^3 + \sum_{m=1}^{k} d \cdot x_m^2$$

$$\sum_{m=1}^{k} y_m \cdot x_m = \sum_{m=1}^{k} a \cdot x_m^4 + \sum_{m=1}^{k} b \cdot x_m^3 + \sum_{m=1}^{k} c \cdot x_m^2 + \sum_{m=1}^{k} d \cdot x_m$$

$$\sum_{m=1}^{k} y_m \cdot x_m^0 = \sum_{m=1}^{k} a \cdot x_m^3 + \sum_{m=1}^{k} b \cdot x_m^2 + \sum_{m=1}^{k} c \cdot x_m + \sum_{m=1}^{k} d \cdot x_m^0$$

We can rearrange these equations in matrix form as $[A] \cdot \text{coef} = \vec{x} \cdot \vec{y}$ where
The polynomial is still given as $P = \sum_{i=1}^{n+1} \text{coef}_i x^{n+1-i}$

The least-squares polynomial fit is invoked in Maple as

\begin{equation}
\text{CurveFitting[LeastSquares]}(xv, yv, H, \text{curve} = a + b \cdot H + c \cdot H^2 + d \cdot H^3 + e \cdot H^4) \nonumber
\end{equation}

\begin{align*}
0.0211188811188832 + 0.00814025835275834 \quad H - 0.0000205179195804196 \quad H^2 + 2.26816239316239 \times 10^{-8} \quad H^3 \\
- 8.98892773892771 \times 10^{12} \quad H^4 
\end{align*}

(9)

$poly2 := \text{plot}((9), H = 0 .. 1.000) ;$

$\text{plots[display]}(\text{points1}, poly2)$
Notice that while the errors are minimized the error magnitudes are still noticeably large so we increase the order of the polynomial until the plot of the least-squares polynomial passes through the required points.

\[
\text{CurveFitting[LeastSquares]}(xv, yv, H, \text{curve} = a + b \cdot H + c \cdot H^2 + d \cdot H^3 + e \cdot H^4 + f \cdot H^5)
\]

\[
0.00429195804195410 + 0.00945930215617723 \times H - 0.0000313853074009327 \times H^2 + 5.32972756410261 \times 10^{-8} \times H^3 \quad (10)
- 4.40450174825178 \times 10^{-11} \times H^4 + 1.40224358974360 \times 10^{-14} \times H^5
\]

\[
poly3 := \text{plot}((10), H=0..1000):
\]

\[
\text{plots[display]}(\text{points1, poly3})
\]
CurveFitting[LeastSquares](xv, yv, H, curve = a + b·H + c·H^2 + d·H^3 + e·H^4 + f·H^5 + g·H^6)
0.000635540929637279 + 0.0101985078157137 H - 0.0000404530187051491 H^2 + 9.22990581722111 10^{-8} H^3
- 1.19407836852673 10^{-10} H^4 + 8.10567496229125 10^{-14} H^5 - 2.23447712418258 10^{-17} H^6

poly4 := plot((11), H=0 .. 1000):

plots[display](points1, poly4)
We judge poly4 (the 6th order polynomial) sufficiently accurate.

With a polynomial function $B(H)$ in hand we are ready to solve the single-path magnetic flux problem both by the loadline method, and by Newton's method. The problem is to find the magnetic flux density $|B_g|$ in the air gap of the silicon sheet steel structure shown in Figure 2 below. (The flux density in the air gap $|B_g|$ is the same as the flux in the core $|B_c|$.)
Our first task is to extract the problem parameters from the drawing of Figure 1. The value of $L_e$ is obviously 1 mm which is the same as 0.001 m. (The dimensions must be in meters for the load-line equation to be correct.)

The value of $L_e$ is a little more difficult to determine. What we need is called the mean-path-length, defined as the average path-length of the flux in the iron. (The dashed line in Figure 1.) The average flux-line travels along the middle of the core, so the horizontal distance is

$$2 \cdot \left( 0.2 - \frac{0.04}{2} - \frac{0.04}{2} \right) = 2 \cdot (0.2 - 0.04) = 2 \cdot (0.16)$$

The vertical distance in the left-most leg is

$$\left( 0.2 - \frac{0.04}{2} - \frac{0.04}{2} \right) = 0.16$$

The vertical distance in the right-most leg is

$$\left( 0.2 - \frac{0.04}{2} - \frac{0.04}{2} - 0.001 \right) = 0.159$$

So the mean-path length of the core is $L_e = 3 \cdot 0.16 + 0.159 = 0.639$

(We could also calculate as $L_e = 4 \cdot 0.16 - 0.001 = 0.639$.)

The relationship between the current $I$ and the flux density $B_g$ is given by the load-line equation and our sixth-order polynomial fit to the BH curve of silicon sheet steel. So we have

$$Bc \Rightarrow H \rightarrow (11)$$
\[ H \rightarrow 0.000635540929637279 + 0.010198507815713664 H - 0.000040453018705149145 H^2 \]
\[ + 9.229905817221113 \times 10^{-8} H^3 - 1.194078368526735 \times 10^{-10} H^4 + 8.105674962291246 \times 10^{-14} H^5 \]
\[ - 2.2344771241825796 \times 10^{-17} H^6 \]

We must change the imaginary operator (unit) so that we may use I for the current variable.

\[ \text{interface( imaginaryunit = p }) \]

\[ I \]

The load-line equation is

\[ B = - \frac{\mu_0 \cdot \frac{L}{g} \cdot H + \frac{\mu_0}{g} \cdot Nt \cdot I}{\frac{L}{g}} - \frac{\mu_0 Nt I}{\frac{L}{g}} \]

\[ \text{eval((14), (15))} \]

\[ B = -0.0002556000000 \pi H + 0.4000000000 \pi \]

\[ \text{loadlineplot} := \text{plot(rhs((16)), H=0..1000)} : \]

\[ \text{BHPplot} := \text{plot}(B, 0..1000) : \]

\[ \text{plots[display]}(\text{loadlineplot, BHPplot}) \]
So our initial estimate is \( H_c = 250 \) and \( B_c = 1.05 \)

\[ B = (11) \]
\[ B = 0.000635540929637279 + 0.0101985078157137 \ H - 0.0000404530187051491 \ H^2 \\
+ 9.22990581722111 \times 10^{-8} \ H^3 - 1.19407836852673 \times 10^{-10} \ H^4 + 8.10567496229125 \times 10^{-14} \ H^5 \\
- 2.23447712418258 \times 10^{-17} \ H^6 \]

\( f_{solve}( ((16), (17)), \{B = 1.05, H = 250\}) \)

\[ \{B = 1.061799759, H = 242.6394353\} \]

Solving Sets of Equations with Newton's Method

First recall our recipe (Newton's Method) for finding solutions to problems that can be written

\[ f(x) = 0 \] for \( x \)
\[ x^{new} = x^{old} - \frac{f(x^{old})}{\frac{d}{dx} f(x) \bigg|_{x=x^{old}}} \]

Several of our analyses have involved solving sets of tightly coupled non-linear equations, which often prompts students to ask, "How does Maple solve SETS of equations using Newton's method?" This leads naturally to the following extension of Newton's method to vector equations. Suppose we have a system of equations:

\[
\begin{align*}
\dot{f}_1(x_1, x_2, \ldots, x_n) &= 0 \\
\dot{f}_2(x_1, x_2, \ldots, x_n) &= 0 \\
& \vdots \\
\dot{f}_n(x_1, x_2, \ldots, x_n) &= 0
\end{align*}
\]

We write the system in matrix form as \( \vec{F}(\vec{x}) = \vec{0} \) where

\[
\vec{F} = \begin{bmatrix}
\dot{f}_1(x_1, x_2, \ldots, x_n) \\
\dot{f}_2(x_1, x_2, \ldots, x_n) \\
& \vdots \\
\dot{f}_n(x_1, x_2, \ldots, x_n)
\end{bmatrix}, \quad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad \vec{0} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}
\]

In order to extend Newton's method

\[
x^{new} = x^{old} - \frac{f(x^{old})}{\frac{d}{dx} f(x) \bigg|_{x=x^{old}}} \]

to vector-valued, vector functions, we must make sense of the derivative of \( \vec{F}(\vec{x}) \) with respect to \( \vec{x} \).

We define derivative of \( \vec{F}(\vec{x}) \) with respect to \( \vec{x} \) as the matrix of all possible partial derivatives of \( \vec{F} \) with respect each of the variables in \( \vec{x} \). This matrix is called the Jacobian matrix (after Jacobi who proposed it) and is defined as

\[
J_{i,j} = \frac{\partial}{\partial x_j} F_i
\]

or
\[
J = \begin{bmatrix}
\frac{\partial}{\partial x_1} F_1 & \frac{\partial}{\partial x_2} F_1 & \cdots & \frac{\partial}{\partial x_n} F_1 \\
\frac{\partial}{\partial x_1} F_2 & \frac{\partial}{\partial x_2} F_2 & \cdots & \frac{\partial}{\partial x_n} F_2 \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial}{\partial x_1} F_n & \frac{\partial}{\partial x_2} F_n & \cdots & \frac{\partial}{\partial x_n} F_n
\end{bmatrix}
\]

Newton's method for vector equations becomes then,

\[
\vec{x}^{(new)} = \vec{x}^{(old)} - \left( J_x \left( \vec{F}(\vec{x}^{(old)}) \right) \right)^{-1} \cdot \vec{F}(\vec{x}^{(old)})
\]

In general we have

\[
\vec{x}^{(n+1)} = \vec{x}^{(n)} - \left( J_x \left( \vec{F}(\vec{x}^{(n)}) \right) \right)^{-1} \cdot \vec{F}(\vec{x}^{(n)})
\]

As an Example consider \( \vec{F} = \begin{bmatrix} (xy^2z)^3 \\ x^2y + xz^3 \\ x^2 + xy + z^2 \end{bmatrix} \) with \( \vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \)

\[
\begin{bmatrix}
(xyz)^3 \\
x^2y + xz^3 \\
x^2 + xy + z^2
\end{bmatrix}
\]

\[
\begin{bmatrix}
x^3y^3z^3 \\
x^2y + xz^3 \\
x^2 + xy + z^2
\end{bmatrix}
\]

(19)

\[
\begin{bmatrix}
x \\ y \\ z
\end{bmatrix}
\]

(20)
\[ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]

(21)

\[ (21)_{i,j} := \frac{d}{d(20)_i} (19)_j \]

end do ;
end do ;

(21)

\[ \begin{bmatrix} 3x^2 y^2 z^3 & 2xy + z^2 & 2x + y \\ 3x^3 y^2 z^3 & x^2 & x \\ 3x^3 y^3 z^2 & 3xz^2 & 2z \end{bmatrix} \]

(22)

### Homework Problems

#1. Fit an interpolating polynomial to the points

\((-2,4), (-1,2), (0,1), (1,4), (2,3), 3,6\)

and plot the points and the polynomial on the same set of axes.

#2. Fit a fourth-order least-squares polynomial to the above points, and plot the points and the polynomial on the same set of axes.

#3. Find the Jacobian of

\[ 6x_1^2 + 3x_1x_2 + 4x_3^2 = 0 \]
\[ \cos(2x_1) + \sin(x_2) = 5x_3 \]
\[ 4x_1^2 + 2x_1 = x_2 \]

#4. Evaluate the above Jacobian at \( x = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \)

#5. Fit an interpolating polynomial and an optimal least-squares polynomial to the magnetization curve for cast steel in the BH curve shown below.
Transformers

If we construct the device shown in Fig. 5-1 we have a pair of mutually coupled coils.

Figure 5-1    Mutually Coupled Coils
The coils are called mutually coupled because the voltage \( V_1 \) depends on both \( I_1 \) and \( I_2 \), and the voltage \( V_2 \) depends on both \( I_2 \) and \( I_1 \). The amount of cross dependence i.e. the amount by which \( V_1 \) depends on \( I_2 \) and \( V_2 \) depends on \( I_1 \) is described by the coupling coefficient \( k \)

\[
k = \frac{M}{\sqrt{L_1 \cdot L_2}}
\]

where \( M \) is the mutual inductance. A pair of mutually coupled coils is a four terminal device whose terminal voltages and currents are described by

\[
V_1 = L_1 \frac{d I_1}{d t} + M \frac{d I_2}{d t}
\]

\[
V_2 = M \frac{d I_1}{d t} + L_2 \frac{d I_2}{d t}
\]

If we imagine a "perfectly coupled" set of coils such that all of the flux produced by the current in coil 1 links coil 2, and all of the flux produced by the current in coil 2 links coil 1 we would have \( k = 1 \), and we call the device an ideal transformer. The assumption that \( k = 1 \) dramatically changes the behavior of the device. Instead of \( V_1 \) depending on the two currents \( I_1 \) and \( I_2 \), \( V_1 \) depends only on \( V_2 \), while \( I_1 \) depends only on \( I_2 \). If we reverse the reference direction of \( I_2 \) as shown in Fig. 5-2 the terminal equations become

\[
V_2 = \frac{N_2}{N_1} \cdot V_1
\]

\[
I_2 = \frac{N_1}{N_2} \cdot I_1
\]

(The transformer in Fig. 5-2 is shown loaded by a resistor \( R_L \))

\[
R_m = \frac{N_1^2}{N_2} \cdot R_L
\]

5-2
Figure 5-2  Ideal Transformer

Now if we build the circuit of Fig. 5-3 with a real, physical transformer, we find that the voltage $V_2$ does not remain constant at $(N_2 / N_1) \cdot V_1$, but we find that $V_2$ decreases as $R_L$ is decreased i.e. as the load is increased. (The terms "increase the load" and "decrease the load" refer to increasing or decreasing the load current. Thus it is the current, not the load resistance that is the reference when discussing loads.) Further, we find that as the load resistance in the circuit of Fig. 5-3 decreases (the load increases), the phase difference between $V_1$ and $V_2$ increases.
Since the secondary voltage of a real transformer is not the secondary voltage predicted by the terminal equations of the ideal transformer it would seem that we should not model real transformers by the ideal transformer equations. However, the ideal transformer is such a useful concept that we desire to see if the ideal transformer model can be modified to take into account our observations concerning the effects of changing the load current. The modification that we need to make to the ideal transformer to account for the changes in $V_2$ with load current, is to add a series impedance $R_{eq} + jX_{eq}$ to the secondary of the ideal transformer as shown in Fig. 5-4.
Figure 5-4 Winding Resistance and Leakage Reactance

Before we accept this as our standard model, let us try another experiment. Let us set the load current in the circuit of Fig. 5-3 to zero ($R_L$ is infinite), and measure $V_1$ and $I_1$. Now we expect $I_2$ and $I_1$ both to be zero since

$$R_{in} = \frac{N_1^2}{N_2} R_L = \infty$$

if $R_L = \infty$. However, when we build the circuit and measure $V_1$ and $I_1$, we find that not only is $I_1$ not equal to zero, but $I_1$ is not in phase with $V_1$. We take this effect into account in our model of the physical transformer by adding an impedance $R_e \parallel X_m$ in parallel with the primary of the transformer as shown in Fig. 5-5.
The circuit model shown in Fig. 5-5 is called the Steinmetz model of the transformer after Karl Steinmetz, the General Electric engineer who first proposed it. We wish to perform some tests on the transformer to determine the parameters of the Steinmetz model. The first test to which we shall subject our physical transformer is the open circuit test, as illustrated in Fig. 5-6. From this test we can calculate the turns ratio \( N_1 / N_2 \), \( R_c \) and \( X_m \). The test is performed by exciting the low voltage winding with rated voltage, and measuring the primary power \( P_{oc} \), the primary current \( I_{oc} \), the primary voltage \( V_{oc} \), and the secondary voltage \( V_{2oc} \). (The open circuit test is always performed by exciting the low voltage winding.)
Figure 5-6  Open Circuit Test

\[
\left(\frac{N_1}{N_2}\right) = \frac{V_{OC}}{V_{2OC}}
\]

The open circuit Steinmetz model parameters are then calculated as

\[
R_e = \frac{V_{OC}^2}{P_{OC}}
\]

\[
\theta_{OC} = \cos^{-1}\left(\frac{P_{OC}}{V_{OC} \cdot I_{OC}}\right)
\]

\[
X_m = \frac{V_{OC}}{I_{OC} \cdot \sin(\theta)}
\]

5-7
$R_{eq}$ and $X_{eq}$ are found from the short circuit test. In this test the transformer's low voltage winding is short circuited and a voltage $V_{sc}$ is applied to the transformer high voltage winding such that rated current flows in the high voltage winding. (See Fig. 5-7)

**Figure 5-7  Short Circuit Test**

$V_{sc}$, $P_{sc}$, and $I_{sc}$ are measured and then $R_{eq}$ and $X_{eq}$ are found from

$$Z_{eq} = \frac{V_{sc}}{I_{sc}}$$

$$\theta_{eq} = \cos^{-1} \left( \frac{P_{sc}}{V_{sc} \cdot I_{sc}} \right)$$

$$\tilde{Z}_{eq} = Z_{eq} \angle \theta_{eq} = R_{eq} + jX_{eq}$$
Tapped windings and multi winding transformers

If connections are made to intermediate turns of a coil and brought out to the outside world the result is called a tapped transformer. This device essentially has several different turns ratios depending on which tap of the transformer is used. This phenomenon can be used to "adjust" the transformer's output voltage for variations in the input voltage, or in the load current. An example is shown in Fig. 5-8 where the output voltage is either 52.5 V, 50 V, or 47.5 V, depending on which tap we connect to the load.

---

Fig 5-8  Tapped Transformer
In fact, there are transformers that can change taps automatically as the input voltage changes.

Some transformers have more than one primary winding and more than one secondary winding. This is done so that several different transformers (with different turns ratios) can be implemented with the same device. For example, if the primary windings of the transformer shown in Fig. 5-9 are wired in series and the secondary windings are placed in series, the turns ratio of the resulting "composite transformer" would be 2:1, its primary voltage rating would be 240 V, its secondary voltage rating would be 120 V, its primary current rating would be $\frac{1}{2}$ A, and its secondary current rating would be 1 A. If the primary windings are placed in parallel, and the secondary windings are placed in parallel, the turns ratio would still be 2:1 But the primary voltage rating would be 120 V, the secondary voltage rating would be 60 V, the primary current rating would be 1 A, and the secondary current rating would be 2 A. Placing the primary windings in series and the secondary winding in parallel would produce a 4:1 turns ratio, and placing the primary winding in parallel and the secondary windings in series would produce a turns ratio of 1:1.

![Diagram of Multi-Winding Transformer]

---

The transformers upon which we shall perform our experiments have the structure shown in Fig. 5-10. (The double vertical line indicates that all the coils are wound on the same iron core.)
Figure 5-10  Lab Experimental Transformer Ratings and Terminal Connections

Notice that the low voltage winding are marked $X_n$ and the high voltage windings are marked $H_r$. The numbers are assigned so that the lowest potential is at the lowest numbered terminal and the highest potential at the highest numbered terminal. These are universally accepted markings.
(National Electric Manufacturers Association or NEMA Standard) and must be used as a guide to wiring the transformer, as this is the arrangement to which the transformer's published specifications apply. In other words if you violate this convention, the manufacturer does not warranty that the transformer will work according to the published specifications.

The designations \( N_a \) - \( N_k \) refer to the number of turns between the indicated terminals. For example the number of turns between H2 and H1 is \( N_{a2} \), between H3 and H2 there are \( N_k \) turns, etc. To find the turns ratio between any two coils, excite one of them with a known voltage and measure the voltage across the other coil. The ratio of the voltages is the turns ratio. For example to find \( \frac{N_f}{N_a} \) we could excite terminals H2 H1 with ten volts and measure \( V_{X3 \times 2} \) then

\[
\frac{N_f}{N_a} = \frac{V_{X4 \times 3}}{10}
\]

(Remember that \( V_{X3 \times 2} = V_{X3} - V_{X2} \))

To use the dot convention, the dot is assumed (by the above convention) to be on the highest numbered terminal of each coil. (One could also produce consistent results by assuming the dot on the lowest numbered terminal of each coil - but you must be consistent.)

The lab single phase transformers are 120 VA devices. Since we assume that the power out equals the power in (almost), we can carry out the calculation for \( |S| \) on the high voltage side as \( |S| = V_{HI \times HI} \) or on the low voltage side as \( |S| = V_{LO \times LO} \). For series operation of the high voltage side we have \( |S| = \frac{1}{2} A \cdot 240 V = 120 VA \), and for parallel operation of the high voltage side we have \( |S| = 1 A \cdot 120 V = 120 VA \).

For the low voltage side series operation gives \( |S| = 1 A \cdot 120 V = 120 VA \), and parallel operation gives \( |S| = 2 A \cdot 60 V = 120 VA \).

If only one primary coil is used (or if only one secondary coil is used) the transformer must be derated to 60 VA.

The transformers in this lab are fused against excessive currents. The most common cause of excessive current is the inrush current that occurs if the transformer input voltage is raised (or lowered) too suddenly. When a transformer is suddenly energized, a large current (the inrush current) must flow to establish the magnetic field necessary to Faraday induction. For this reason it is VERY important to ALWAYS CHANGE THE VOLTAGE APPLIED TO A TRANSFORMER SLOWLY to avoid large inrush currents that can blow the fuse. (The fuse is a real nuisance to replace.)

Experiment 5. Single phase transformers (2 weeks)

1. With the 0 - 240/140 VAC breaker open, construct the circuit shown below (Turns Ratio
Test). Remember to increase the voltage to the transformer SLOWLY to avoid blowing the fuse - NEVER set the supply to 100 V and then use the breaker to make the connection to the transformer. ALWAYS set the voltage to zero, close the breaker, then set the voltage to the desired level. Measure and record:

\[ V_{h3\ h1}, V_{h3\ h2}, V_{h2\ h1}, V_{X4\ X3}, V_{X4\ X2}, V_{X3\ X2}, V_{X3\ X1}, V_{X2\ X1}, V_{X7\ X6}, V_{X7\ X5}, V_{X6\ X5} \]

2. Deenergize the transformer (do not open the breaker on an energized transformer - set the voltage to zero then open the breaker) and connect X4 to X5. Reenergize the transformer, (see above procedure) and measure \( V_{X7\ X1} \).

3. Deenergize the transformer (as above), remove the X4 to X5 connection, connect X4 to X7, reenergize the transformer, and measure \( V_{X5\ X1} \).

Since both of these measurements are the voltage of coils X1X4 and X5X7 in series, explain why the results are different.
4. With the 0 - 240/140 VAC breaker open, construct the circuit shown below (Open Circuit Test). The transformer we create with this arrangement has the high voltage windings in series and the low voltage windings in parallel. We shall use this transformer for the most of the exercises in this lab.
Open Circuit Test

5. Close the 0 - 240/140 VAC breaker and bring the voltage on the low voltage winding to its rated value. Measure and record $V_{OC}$, $I_{OC}$, and $P_{OC}$ using the V, I, and P buttons on the wattmeter. Ensure that

$$P_{OC} < V_{OC} \cdot I_{OC}$$

6. Construct the circuit shown below (Short Circuit Test).
0 - 240/140 VAC

Short-circuit Test

7. Close the 0 - 240/140 VAC breaker and increase the voltage until rated current is flowing in the high voltage winding. Measure and record $V_{SC}$, $I_{SC}$, and $P_{SC}$ using the V, I, and P buttons on the wattmeter. Check to see that

$$ P_{SC} < V_{SC} \cdot I_{SC} $$

8. Construct the circuit shown below (Load Test).
Load Test

11. With all load resistor switches off, close the 0 - 240/140 VAC breaker, and increase the voltage until rated voltage appears across H5 H1. Now close enough switches on the RL-100A until rated current flows from the low voltage windings, and measure and record $V_{H5H1}$, $V_{X4X1}$, and $I_L$.

12. Connect the circuit shown below.
12. Set $R = 333.333 \, \Omega$, use phase a reactive load with the knob set midway towards full lag for the adjustable load reactance, and use phase b and phase c reactive loads connected in parallel (set the knobs to their mid-position [zero capacitance] for now) to form the capacitor.

13. Close the supply breaker, increase the supply voltage to 120 V, and record the supply voltage and the load voltage.

14. Increase the capacitance until the load voltage = the supply voltage (or the load voltage is as high as it will go), and record the load voltage and the capacitor current.

16. Using the results of 1. calculate the turns ratios:

$$\frac{N_a}{N_c}, \frac{N_b}{N_c}, \frac{N_d}{N_c}, \frac{N_e}{N_c}, \frac{N_f}{N_c}, \frac{N_g}{N_c} \text{, and } \frac{N_k}{N_c}$$

17. Using the results of 4. through 9. find the Steinmetz model of the transformer you created by placing the high voltage windings in series and the low voltage windings in parallel. (Different winding arrangements would have different Steinmetz models.)
18. Use your Steinmetz model to predict the load voltage of part 10. and 11. and compare to your experimental results.

19. Calculate the capacitance you have added in part 14. and analyze the circuit to calculate the load voltage. Compare you calculation to your measurement.
Three-Phase Transformers

A balanced system of three phase-voltages at one voltage level (line voltage) can be transformed to another balanced system of three-phase voltages at a different voltage level (line voltage) by a three-phase transformer. A three-phase transformer is simply a bank of three single-phase transformers with their windings connected in a specific way. A collection of three single phase transformers have a total of 12 terminals: six primary terminals and six secondary terminals. The six primary windings can be either Y connected or Δ connected, and the six secondary windings can be either Y connected or Δ connected. There are then four different configurations of three-phase transformers: YY, ΔΔ, YΔ, and ΔY. (See Fig. 6-1 for a YΔ transformer.)

![Diagram of a YΔ three-phase transformer](image)

**Fig. 6-1**  
**YΔ Three-Phase Transformer**

In Fig. 6-1, the primary phase connections are indicated by lower case single letters (a, b, c), and the primary coil terminals are indicated by lower case double letters (aa aa' are the terminals of one of the primary coils). The secondary phase connections are indicated by upper case single letters (A, B, C), and the secondary coil terminals are indicated by upper case double letters (AA and AA' are the terminals of one of the secondary coils). As Fig. 6-1 demonstrates, following the connections on a schematic or wiring diagram can be quite confusing, so the simplified diagram shown in Fig. 6-2 is often used.
Fig. 6-2  
Simplified YΔ Representation

The interpretation of Fig. 6-2 is that each line represents a transformer coil, and lines that are parallel represent coils of the same transformer.

Consider the YY connected three-phase transformer shown in Fig. 6-3.
If we excite the primary with a balanced system of three phase voltages with voltage $V$ (remember $V$ is the RMS value of the line voltage), and each transformer is a turns ratio of $n:1$, then the secondary line voltage is calculated as follows. Across each primary coil (aa aa' for example) we have a voltage of \[ \frac{V}{\sqrt{3}} \] which means that across each secondary coil (AA AA" for example) we have a voltage of \[ \frac{V}{n \cdot \sqrt{3}}. \] The secondary line voltage then is \[ \frac{\sqrt{3} \cdot V}{n \cdot \sqrt{3}} = \frac{V}{n}, \] and the primary and secondary line voltages are related as $n:1$, and since both primary and secondary voltages are phase voltages, the secondary voltage is in phase with the primary voltage. A similar analysis for the \( \Delta \Delta \) (Fig. 6-5) connection gives that the line voltages are again related as $n:1$, and that the secondary voltage is in phase with the primary voltage since both voltages are line voltages. The results for the other two connections (\( Y \Delta \) and \( \Delta Y \)) are not so simple.

Consider the \( Y \Delta \) connected transformer of Fig. 6-2. If we excite the primary with a three-phase voltage of $V$, then each primary coil has voltage \[ \frac{V}{\sqrt{3}} \] as before. Each secondary coil then has a voltage of \[ \frac{V}{n \cdot \sqrt{3}}, \] but the secondary coils are connected across the lines. Therefore the voltage \[ \frac{V}{n \cdot \sqrt{3}} \] is the line voltage (not the phase voltage) and the primary and secondary line voltages are not related as $n:1$ but as \( n \cdot \sqrt{3} : 1 \). This is true since the primary line voltage divided by the secondary line voltage is

\[ \frac{V}{\left( \frac{V}{n \cdot \sqrt{3}} \right)} = n \cdot \sqrt{3} \]

In addition, since the primary voltage is a phase voltage, and since the transformer secondary voltage is in phase with the transformer primary voltage, the secondary line voltage is in phase with the primary phase voltage. Therefore, the secondary line voltage lags the primary line voltage by $30^\circ$.

A similar analysis of the \( \Delta Y \) connection of Fig. 6.4 (on page 6-4) gives that the primary and secondary line voltages are related by \[ \frac{n}{\sqrt{3}} : 1, \] and that the secondary line voltage leads the
primary line voltage by 30°. Thus the same bank of three single phase transformers can give three line different line voltage ratios, (and three different phase relationships) depending on how they are connected.

Fig. 6-4        Simplified Δ Y Representation

The Δ Δ connection of Fig. 6-5 has the interesting property that removing one of the transformers from the three-phase bank has absolutely no change on the secondary voltages if the transformer bank is not loaded. (i.e. if no load current is flowing.)

Fig. 6-5        Simplified Δ Δ Representation
If one removes one of the transformers in a Δ Δ connected three-phase bank the resulting configuration is called an open Δ or V connection, and is shown in Fig. 6-6.

![Diagram](image)

**Fig. 6-6  Simplified Open Δ (or V) Connection**

Let us calculate the secondary voltages. $V_{AB} = nV_{ab}$ and $V_{BC} = nV_{bc}$ as usual. Then by KVL, $V_{CA} = V_{CB} + V_{BA} = nV(-1 \angle -90^\circ + -1 \angle 30^\circ) = nV(1 \angle 150^\circ)$ (See Fig. 6-7)

So we see that the unloaded open Δ has the same line voltages as the unloaded Δ Δ connection.

![Diagram](image)

**Fig.6-7  3 Φ Vector Voltages**

6-5
When a Δ connected secondary is delivering full power to a three-phase load (Fig. 6-8), we have

\[ P_\Delta = \sqrt{3} \ V_{LINE} \ I_{LINE} \ \cos(\theta) = \sqrt{3} \ V_{RATED} \left(\sqrt{3} \ I_{RATED}\right) \ \cos(\theta) \]

since the line current is \( \sqrt{3} \) times the phase (winding) current.

![Diagram of Loaded Δ Connected Secondary](image)

**Fig. 6-8**  Loaded Δ Connected Secondary

When a V connected (open Δ) secondary is supplying rated power to a three-phase load (Fig. 6-9) we have:

\[ P_V = \vec{V}_{AB} \cdot \vec{I}_A + \vec{V}_{CB} \cdot \vec{I}_C \]

![Diagram of Loaded V Connected Secondary](image)

**Fig. 6-9**  Loaded V Connected Secondary

6-6
From Fig. 6-10, we have
\[\vec{V}_{AB} \vec{I}_A = V_{AB} I_A \cos(\phi+30^\circ)\] and \[\vec{V}_{CB} \vec{I}_C = V_{CB} I_C \cos(\phi-30^\circ)\]

Fig. 6-10 3 Φ Vector Currents

Since the transformer is delivering full rated power we have
\[V_{AB} = V_{CB} = V_{RATED}\] and \[I_A = I_C = I_{RATED}\]

The power delivered by a fully loaded V connected transformer is then
\[P_V = V_{RATED} I_{RATED} (\cos(30^\circ + \theta) + \cos(30^\circ - \theta))\]
By trigonometric substitution we have

\[ P_V = V_{RATED} I_{RATED} \left( \cos(30^\circ) \cos(\theta) - \sin(30^\circ) \sin(\theta) + \cos(30^\circ) \cos(\theta) + \sin(30^\circ) \sin(\theta) \right) \]

or

\[ P_V = 2 \ V_{RATED} I_{RATED} \cos(30^\circ) \cos(\theta) \]

So a three phase \( \Delta \) connected transformer whose kVA rating is \( S_\Delta \) is only capable of producing \( \frac{P_V}{P_\Delta} S_\Delta \). The transformer must be derated for use in a V connection by

\[ \frac{P_V}{P_\Delta} = \frac{2 \ V_{RATED} I_{RATED} \cos(\theta) \cos(30^\circ)}{3 \ V_{RATED} I_{RATED} \cos(\theta)} \]

or

\[ \frac{P_V}{P_\Delta} = \frac{2 \cos(30^\circ)}{3} = \frac{1}{\sqrt{3}} = 0.5774 \]

So a \( \Delta \) connected transformer bank must be derated to 58% of its three-phase rating if one of the transformers is removed to form a V connection. Note that this is less that \( 2/3 \) of the original rating.
Experiment 6. Three-Phase Transformers. (2 weeks)

1. Determine the turns ratio of each of the three transformers in the three-phase transformer bank. (The transformers are each 100 VA with a rated primary voltage of 115 V.)

2. Connect the three transformers as a YY bank as shown below. Set the three-phase LINE TO LINE voltage on the primary side to 140 V, trigger the scope from phase a (see experiment 2. step 4 - 6.), and measure and record $\bar{V}_a$ and $\bar{V}_{ab}$ and $\bar{V}_a$ and $\bar{V}_{AB}$ preserving their phase relationship. What is the apparent turns ratio?
3. Change the secondary to a \( \Delta \) connection as shown below. With the same primary excitation as above, measure and record \( \tilde{V}_a \) and \( \tilde{V}_{ab} \) and \( \tilde{V}_a \) and \( \tilde{V}_{AB} \). What is the apparent turns ratio?

\[
\begin{array}{c}
\begin{array}{c}
\text{a} \\
\text{b} \\
\text{c} \\
n
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
A \\
B \\
C
\end{array}
\end{array}
\]

\(0 - 240/140 \text{ VAC}\)

4. Leave the \( \Delta \) connection in the secondary, change the primary connection to a \( \Delta \) configuration, and using the same primary excitation as above measure and record \( \tilde{V}_a \) and \( \tilde{V}_{ab} \) and \( \tilde{V}_a \) and \( \tilde{V}_{AB} \). What is the apparent turns ratio?

Display \( \tilde{V}_{AB} \) and \( \tilde{V}_{BC} \) together, and \( \tilde{V}_{AB} \) and \( \tilde{V}_{CA} \) together.

5. Leave the \( \Delta \) connection in the secondary and in the primary, and remove the transformer connected between a and c (and A and C) to form a \( \text{Y} \) connection, and using the same primary excitation as above measure and record \( \tilde{V}_a \) and \( \tilde{V}_{ab} \) and \( \tilde{V}_a \) and \( \tilde{V}_{AB} \). What is the apparent turns ratio?

Display \( \tilde{V}_{AB} \) and \( \tilde{V}_{BC} \) together, and \( \tilde{V}_{AB} \) and \( \tilde{V}_{CA} \) together.

6. Replace the missing transformer, leave the \( \Delta \) connection in the primary, change the secondary to a \( \text{Y} \) connection, and using the same primary excitation as above measure and record \( \tilde{V}_a \) and \( \tilde{V}_{ab} \) and \( \tilde{V}_a \) and \( \tilde{V}_{AB} \). What is the apparent turns ratio?
7. Return to the Δ Δ connection of step 4. and load the secondary with a 333 Ω, Y connected, three-phase resistive load. Using the same primary excitation as above, measure all three line currents and all three line voltages. (Magnitudes only)

8. Return to the V connection and, leaving the load in place, measure all three line currents and all three line voltages (magnitudes only), using the same primary excitation as above.

9. Compare each of your observations to the theoretical prediction.
Transformer Design

We seek to design a transformer (i.e. specify the number of turns and diameter of wire for the primary and secondary windings, and specify the dimensions of the core) given the primary voltage $E_p$, the secondary voltage $E_s$, and the rated apparent power (rated KVA) $S_r$. If we assume a turns ratio of $n:1$ we have:

$$E_p = nE_s$$

$$I_s = nI_p$$

and

$$S_r = E_pI_p = E_sI_s$$

from which we can calculate the primary and secondary rated currents as

$$I_{P\text{RATED}} = \frac{S_r}{E_p}$$

$$I_{S\text{RATED}} = \frac{S_r}{E_s}$$

Now the resistance of wire depends on the cross-sectional area of the wire: larger wire has smaller resistance, smaller wire has larger resistance. Since the copper losses ($I^2R$ losses) of the transformer depend on the resistance of the wire, the diameter of the wire is an important design consideration. We shall begin by considering the “C” core transformer shown in Fig. 7-1.

![C Core Transformer Diagram]

Fig. 7-1  “C” Core Transformer

The device is called a “C” core transformer because the two halves of the core on which the transformer is wound resemble the letter “C”. ($A_w$ is the area of the “window”.)

In Fig.7-1 only a few turns in a single layer are shown on both the primary and secondary, for
ease of visualization. In a real transformer, there are many (tens or hundreds) of turns wound in several layers. The difficulty is that we require larger diameter wire for larger currents, and we require more turns for higher voltages (see Eq. 7.8), but all the turns (both primary and secondary) must fit in the window!

The voltage induced into an n turn coil is

\[ e = n \frac{d \phi(t)}{dt} \]  \hspace{1cm} (7.4)

If \( \phi(t) \) is sinusoidal, i.e. if

\[ \phi(t) = \Phi_{\text{Max}} \sin(\omega t) \]  \hspace{1cm} (7.5)

then the induced voltage is

\[ e = n \frac{d \phi(t)}{dt} = n \Phi_{\text{Max}} \omega \cos(\omega t) \]  \hspace{1cm} (7.6)

which is sinusoidal with RMS value

\[ E_{\text{RMS}} = \frac{n \omega \Phi_{\text{Max}}}{\sqrt{2}} = \frac{n \omega B_{\text{Max}}}{\sqrt{2}} A_c \]  \hspace{1cm} (7.7)

since the flux \( \phi \) is the flux density \( B \) times the cross-sectional area \( A_c \) of the core.

Now if the number of turns on the primary is \( n_p \) and the number of turns on the secondary is \( n_s \), we have

\[ E_p = \frac{n_p \omega}{\sqrt{2}} B_{\text{Max}} A_c \hspace{1cm} E_s = \frac{n_s \omega}{\sqrt{2}} B_{\text{Max}} A_c \]  \hspace{1cm} (7.8)

Since the voltage depends on the cross-sectional area of the core, we see that in addition to the area of the window, the cross-sectional area of the core is an important design consideration. In fact we shall shortly derive a figure of merit for transformer cores in terms of these two areas. First, however, we must consider two preliminary topics: the maximum flux \( B_{\text{Max}} \) and the current density in the windings.

Typical magnetic materials used for transformer construction exhibit saturation in their B H characteristics as shown in Fig. 7-2 below.
The knee of the B-H curve of the material to be used in the construction of the core is taken as $B_{\text{Max}}$. (Often this value is specified by the manufacturer of the core material.)

In order to keep the transformer temperature uniform we design the transformer to have the same current density in the primary winding as in the secondary winding. In fact, this concept (constant current density) is so important to the design technique we are going to use, that it might be called the fundamental transformer design criterion. It is important to keep the transformer temperature as uniform as possible to maximize the rate of heat removal, as we shall discuss later on. The current density is symbolized $J$ and has units $\frac{\text{Amp}}{m^2}$.

One of the consequences of requiring the current density in the primary winding $J_{\text{pri}}$ to be the same as the current density in the secondary winding $J_{\text{sec}}$ is that the fraction of the window area devoted to primary conductors is the same as the fraction of the window area devoted to secondary conductors. Or

$$J_{\text{pri}} = J_{\text{sec}} \Rightarrow \text{Area}_{\text{pri}} = \text{Area}_{\text{sec}} = \frac{A_w}{2}$$  \hspace{1cm} (7.9)

In order to get a better grasp on this phenomenon, consider the window of the 2:1 transformer shown in Fig. 7-3. Assume that we have just two primary turns, one secondary turn, conductors with rectangular cross section and perfect (zero thickness) insulators sheathing the conductors. Since $n = 2$, the secondary current is twice the primary current (by Eq. (7.1)), but if the current density (in Amperes per square meter) in the two winding is to be the same, then we must have that the area of the secondary conductor is twice the area of the primary conductor as shown in Fig. 7-3.
Area of one Primary Conductor

Area of one Secondary Conductor

Area of one Primary Conductor

Fig. 7-3 Window of 2:1 Transformer with Constant Current Density

We can see that the area of a primary conductor $CA_p$ is $\frac{A_W}{4}$ and that the area of a secondary conductor $CA_s$ is $\frac{A_W}{2}$. Since in this case, and $n_p = 2$ and $n_s = 1$ we have

$$CA_p = \frac{A_W}{2 n_p}, \quad CA_s = \frac{A_W}{2 n_s} \quad (7.10)$$

Further, since $I_p = J_p CA_p$, $I_s = J_s CA_s$, and $J_p = J_s = J$, we have

$$I_p = \frac{JA_W}{2 n_p}, \quad I_s = \frac{JA_W}{2 n_s} \quad (7.11)$$

You should convince yourself by examination of Figs. 7-4 through 7-6 that (7.10) and (7.11) are true in general, i.e. for any values of $n_p$ and $n_s$. 

7-4
Fig. 7-4  Window of 3:1 Transformer

Fig. 7-5  Window of 3:2 Transformer
<table>
<thead>
<tr>
<th>$A_W$</th>
<th>Area of Primary Conductor</th>
<th>Area of Secondary Conductor</th>
<th>$A_W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td></td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>$A_W$</td>
<td>Area of Primary Conductor</td>
<td>Area of Secondary Conductor</td>
<td>$A_W$</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>$A_W$</td>
<td>Area of Primary Conductor</td>
<td>Area of Secondary Conductor</td>
<td>$A_W$</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>$A_W$</td>
<td>Area of Primary Conductor</td>
<td>Area of Secondary Conductor</td>
<td>$A_W$</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td>6</td>
</tr>
</tbody>
</table>

Fig. 7-6 Window of 5:3 Transformer

Unfortunately, wire does not come in rectangular dimensions calculated to fit precisely into the window area, and electrical insulation has finite thickness. When we try to fill a window with real, round wires we have a situation more like that of Fig. 7-7. We seek a way to modify (7.10) and (7.11) to reflect this reality. Let us define window utilization factor $k_u$ such that

$$k_u = \frac{A_{cu}}{A_W}$$

(7.12)

where $A_{cu}$ is the area actually occupied by conductor material (copper). The value of $k_u$ typically varies between 0.3 and 0.5 and is almost universally taken to be 0.4 in design calculations.

Fig. 7-7 Window Utilization Factor
With $k_u$ defined by (7.12), (7.10) becomes
\[
CA_p = \frac{k_u A_w}{2 n_p} \quad \quad \quad CA_s = \frac{k_u A_w}{2 n_s}
\] (7.13)

and (7.11) becomes
\[
I_p = \frac{J k_u A_w}{2 n_p} \quad \quad \quad I_s = \frac{J k_u A_w}{2 n_s}
\] (7.14)

**AREA PRODUCT**

We are now in a position to define the most important design descriptor, the area product $A_p$.

First we write
\[
S_p = E_p I_p
\] (7.15)

Then we substitute for $E_p$ from (7.8), and for $I_p$ from (7.14) which gives
\[
S_p = \frac{n_p \omega}{\sqrt{2}} B_{\text{Max}} A_c \frac{J k_u A_w}{2 n_p}
\] (7.16)

Notice that the $n_p$ in the numerator cancels the $n_p$ in the denominator giving
\[
S_p = \frac{\omega}{\sqrt{2}} B_{\text{Max}} A_c \frac{J k_u A_w}{2}
\] (7.17)

A similar derivation on the secondary side gives
\[
S_s = \frac{n_s \omega}{\sqrt{2}} B_{\text{Max}} A_c \frac{J k_u A_w}{2 n_s} = \frac{\omega}{\sqrt{2}} B_{\text{Max}} A_c \frac{J k_u A_w}{2}
\] (7.18)

Then (recall (7.2))
\[
2 S_r = S_p + S_s = \frac{\omega}{\sqrt{2}} B_{\text{Max}} A_c J A_w
\] (7.19)

Let us define the Area Product $A_p$ as
\[
A_p = A_c A_w
\] (7.20)

Then solving (7.19) for $(A_c)(A_w) = A_p$, we have
\[
A_c A_w = A_p = \frac{2 \sqrt{2} S_r}{\omega B_{\text{Max}} J k_u}
\] (7.21)

where $A_p$ has units (meters$^2$) (meters$^2$) = meters$^4$.

7-7
HEAT DISSIPATION
The total of the iron loss power (hysteresis and eddy current losses) and copper loss power ($I^2 R$) in the primary and secondary windings is dissipated as heat in the transformer, thereby raising the transformer's temperature. Since both the magnetic properties of the core, and the insulation properties of the insulation on the wires degrade as the temperature rises, it is very important to keep the transformer temperature below some maximum temperature. Thus, we shall select the transformer temperature as our next design criterion. Now heat is transferred from the transformer to the ambient atmosphere (the transformer temperature will, by design, always be higher than that of the surrounding air) by radiation and by convection. Heat transfer by radiation is described by the Stefan-Boltzmann law as

$$P_{Rad} = 5.7 \cdot 10^{-8} E A_s \left( T_s^4 - T_a^4 \right)$$  \hspace{1cm} (7.22)

where E is the emissivity of the surface, $A_s$ is the surface area of the object, $T_s$ is the temperature of the object in °K, and $T_a$ is the ambient temperature in °K (the temperature of the surrounding air). For dark objects like black anodized aluminum or black painted objects, the emissivity E is approximately 0.9. For polished aluminum, E may be as small as 0.05, which is why virtually all high power transformers are black. For objects whose vertical height $d_{vert}$ is less than about 1 m, heat loss by convection is given in [1] as approximately

$$P_{Conv} = 1.34 A_s \frac{(T_s - T_a)^{5/4}}{d_{vert}^{1/4}}$$  \hspace{1cm} (7.23)

where $T_s$ and $T_a$ are given in °K (or °C), and $A_s$ is the surface area.

We will use (7.22) and (7.23) to determine the power that can be transferred as heat to the surrounding air, and set that equal to the sum of the iron losses and the copper losses.

In order to do this we must be able to
1. Express the surface area of the transformer in terms of its area product.
2. Express $d_{vert}$ in terms of the area product.
3. Express the iron and copper losses in terms of $J$ and the area product.

We examine the surface area first.

McLyman, working at the Jet Propulsion Laboratory has examined over 12,000 magnetic cores, and tabulated the results of his experiments in [2]. What McLyman found was that the surface area of a transformer is directly proportional to the square root of the area product, and that the constant of proportionality is the same for every "C" core transformer, regardless of its size.

That is we may write

$$A_s = k_s \sqrt{A_p}$$  \hspace{1cm} (7.24)

where $k_s = 39.2$. Eq (7.24) should seem reasonable, since the surface area $A_s$ has units m$^2$ and the area product $A_p$ has units m$^4$. A fully wound "C" core transformer is shown if Fig. 7-8 along with the dimensions necessary to calculate the surface area directly, which we would need if we desired to check the validity of (2.24). (We will not do so here - we will simply accept (7.24).)
We investigate \( d_{\text{vert}} \) by first writing the transformer volume in terms of \( A_p \). McLyman’s results show that

\[
Vol = k_v \left( A_p \right)^{3/4}
\]  

where \( k_v = 17.9 \). You should convince yourself that the units in (7.25) are correct. If we assume a roughly cubical transformer volume then the vertical height \( d_{\text{vert}} \) is given as (see Fig. 7-9)

\[
d_{\text{vert}} = \left( k_v \left( A_p \right)^{3/4} \right)^{1/3} = \left( k_v \right)^{1/3} \left( A_p \right)^{1/4}
\]  

(7.26)
We can calculate the total power that can be dissipated by the transformer to the surrounding air by adding (7.22) and (7.23) and substituting for \( A_i \) from (7.24) and for \( d_{\text{vent}} \) from (7.26) which gives

\[
P_{\text{Rad}+P_{\text{Conv}}} = 5.7 \cdot 10^{-8} E \left( T_s^4 - T_a^4 \right) k_s A_p^2 + \frac{1}{2} \left( \frac{1}{3} \right) \frac{1}{4} \left( \frac{1}{4} \right) k_s A_p^2 \left( T_s - T_a \right)^4 k_s A_p^2 \]

(7.27)

or

\[
P_{\text{Rad}+P_{\text{Conv}}} = 5.7 \cdot 10^{-8} E \left( T_s^4 - T_a^4 \right) k_s A_p^2 + 1.34 \left( T_s - T_a \right)^4 k_s \frac{1}{2} A_p^{12} \frac{1}{16} \]

(7.28)

COPPER LOSSES

The \( I^2 R \) losses in the windings are given as

\[
P_{\text{cu}} = I_p^2 R_p + I_s^2 R_s \]

(7.29)

Substituting for \( I_p \) and \( I_s \) in (7.29) with the expressions given in (7.14) we have

\[
P_{\text{cu}} = \left( \frac{K_u A_w J}{2 n_p} \right)^2 R_p + \left( \frac{K_u A_w J}{2 n_s} \right)^2 R_s \]

(7.30)

In order to write \( R_p \) and \( R_s \) in terms of the area product, we begin with the definition of the resistance of a conductor.

\[
R = \frac{\rho l}{A} \]

(7.31)

We determine the area of the primary and secondary conductors from (7.13) and define the Mean Turn Length MTL as the length of an average turn. (Turns closest to the core will be shorter than the MTL, and turns farther from the core will be longer than the MTL.) Substituting (7.13) and our definition of the MTL into (7.31) we have

\[
R_p = \frac{\text{MTL} n_p \rho}{k_u A_w} = \frac{2 \text{MTL} n_p^2 \rho}{k_u A_w} \]

(7.32)
and

\[ R_s = \frac{MTL \, n_s \, \rho}{2 \, k_u \, A_w} = \frac{2 \, MTL \, n_s^2 \, \rho}{k_u \, A_w} \] (7.33)

Substituting (7.32) and (7.33) into (7.30) we have

\[ P_{cu} = \left( \frac{K_u \, A_w \, J}{2 \, n_p} \right)^2 \frac{2 \, MTL \, n_p^2 \, \rho}{k_u \, A_w} + \left( \frac{K_u \, A_w \, J}{2 \, n_s} \right)^2 \frac{2 \, MTL \, n_s^2 \, \rho}{k_u \, A_w} \] (7.34)

or after some cancellations

\[ P_{cu} = \frac{K_u \, A_w \, J^2}{2} \, MTL \, \rho + \frac{K_u \, A_w \, J^2}{2} \, MTL \, \rho \] (7.35)

or carrying out the addition we have

\[ P_{cu} = K_u \, J^2 \, \rho \, MTL \, A_w \] (7.36)

We must now specify MTL and \( A_w \) in terms of the area product \( A_p \). In order to simplify the design, we will consider only cores with square cross-section and square windows as in Fig. 7-10.

![Fig. 7-10 “C” Core with Square Cross-Section and Square Window](image-url)

7-11
Before proceeding any further, we should point out that the task of specifying individual core dimensions is usually treated as an optimization problem where the design is optimized for weight, cost or volume. These procedures are well beyond the scope of this course and will not be mentioned after this. Taking our simplifying assumptions into account (square cross-section and square window) we can easily express the area product in terms of $a$ and $k$ as

$$A_p = (a)^2 (k a)^2 = k^2 a^4$$  \hspace{1cm} (7.37)$$

which is easily solved for $a$ as $a = \frac{(A_p)^{1/4}}{(k)^{1/2}}$ from which

$$A_w = k^2 a^2 = k^2 \frac{A_p^{1/2}}{k} = k \left(A_p\right)^{1/2}$$  \hspace{1cm} (7.38)$$

To find the MTL consider the top view of the "C" core transformer shown in Fig. 7-11.

![Diagram of "C" core transformer](image)

**Fig. 7-11** Top View of "C" Core Transformer

Examination of Fig. 7-11 reveals that the MTL can be written in terms of $k$ and $a$ as

7-12
\[ MTL = 4 \left( a + \frac{ka}{4} + \frac{ka}{4} \right) = 4 \left( a + \frac{ka}{2} \right) = (4 + 2k)a \quad (7.39) \]

Substituting in (7.39) for \( a \) from the solution of (7.37), we have

\[ MTL = (4 + 2k) \frac{A_p^{\frac{1}{4}}}{\frac{1}{k^2}} = \left( 4k^2 + 2k^2 \right) A_p^{\frac{1}{4}} \quad (7.40) \]

Combining (7.38) with (7.40) we have

\[ MTL A_w = \left( 4k^2 + 2k^2 \right) A_p^{\frac{1}{4}} A_p^{\frac{1}{2}} = \left( 4k^2 + 2k^2 \right) A_p^{\frac{3}{4}} \quad (7.41) \]

Substituting into (7.36) for MTL \( A_w \) from (7.41) we have

\[ P_{cu} = k_u J^2 \rho \left( 4k^2 + 2k^2 \right) A_p^{\frac{3}{4}} \quad (7.42) \]

Now \( \rho \) is a function of temperature \([3]\) given as

\[ \rho|_{T_s} = \rho|_{293^\circ K} \left( 1 + \frac{T_s - 293^\circ}{\frac{1}{\alpha_T} + (T_s - 293^\circ)} \right) \quad (7.43) \]

where \( \alpha_T = 0.00393 \frac{1}{^\circ K} \) and \( \rho|_{293^\circ K} = 1.7241 \cdot 10^{-8} \, \Omega \, m \).

Substituting into (7.42) for the temperature dependence of \( \rho \) from (7.43) we have

\[ P_{cu} = k_u J^2 \rho|_{293^\circ K} \left( 1 + \frac{T_s - 293^\circ}{\frac{1}{\alpha_T} + (T_s - 293^\circ)} \right) \left( 4k^2 + 2k^2 \right) A_p^{\frac{3}{4}} \quad (7.44) \]
IRON LOSSES

The hysteresis losses are given by Steinmetz as

\[ P_h = k_h f B_{Max}^x \left( \text{Volume}_{Core} \right) \]  \hspace{1cm} (7.45)

where the Steinmetz exponent x typically lies between 1.4 and 1.8, and is universally assumed to be x = 1.6 in the absence of better information from the core material manufacturer. The eddy current losses are approximately

\[ P_e = k_e f^2 t^2 B_{Max}^2 \left( \text{Volume}_{Core} \right) \]  \hspace{1cm} (7.46)

where t is the thickness (in meters) of the lamination which is taken to be 0.4 mm in the absence of other information. The constants \( k_h \) and \( k_e \) are obtained from the manufacturer of the core material. (See Appendix II for more information on obtaining these constants.) The volume of the core can be written in terms of \( A_p \), as we will show presently.

The total iron loss in the core is found by adding (7.45) and (7.46) which gives

\[ P_h + P_e = \left( k_h f B_{Max}^x + k_e f^2 t^2 B_{Max}^2 \right) \left( \text{Volume}_{Core} \right) \]  \hspace{1cm} (7.47)

The core volume can be written in terms of a and k (see Fig. 7-10 on page 7-11) in the following way.

\[ \text{Volume}_{Core} = a^2 \left( 2k + 2 \left( ka + 2a \right) \right) = a^3 \left( 4k + 4 \right) \]  \hspace{1cm} (7.48)

Substituting for a in (7.48) with the solution of (7.37) for a, we have

\[ \text{Volume}_{Core} = a^3 \left( 4k + 4 \right) = \left( \frac{1}{4} \right)^3 \left( \frac{A_p}{k^2} \right)^3 \left( 4k + 4 \right) \]  \hspace{1cm} (7.49)

Eq (7.47) then becomes

\[ P_h + P_e = \left( k_h f B_{Max}^x + k_e f^2 t^2 B_{Max}^2 \right) \left( 4k^2 + 4k^2 \right)^3 \]  \hspace{1cm} (7.50)

7-14
We now have what we need to begin the formulation of the design equations. First we set the total transformer losses $P_{cu} + P_n + P_e$ equal to the power that the transformer can dissipate into the surrounding air $P_{Rad} + P_{Conv}$. Next we set the iron losses $P_n + P_e$ equal to the copper losses $P_{cu}$ for maximum efficiency. Finally we invoke (7.21) the definition of the area product. This gives us three equations which must be solved simultaneously for the three unknowns $A_p$, $J$, and $k$. The design equation for the first condition is found by combining (7.28), (7.44), and (7.50) as shown on page 7-16, which is the Maple solution of the first three design equations for $A_p$, $J$ and $k$. The equation for the second condition is found by equating (7.44) to (7.50) as also shown on page 7-16.

The Maple solution is based on the following example:

M6 grain oriented electrical steel is used for the core, and has the following properties

$B_{Max} = 1.5 \text{T} \quad k_h = 51.0 \quad x = 1.6 \quad k_e = 5.9 \cdot 10^6 \quad t = 0.35 \text{ mm}$

$S_r = 10 \text{ KVA} \quad f = 60 \text{ Hz} \quad E_p = 480 \text{ V} \quad E_s = 220 \text{ V}$

$T_a = 20^\circ \text{ C} \quad T_s = 45^\circ \text{ C}$
Solution of the Transformer Design Equations for k, I, and A_p

\[ \text{Cons} := T_a = 273.15 + 20, \quad k_{rad} = 5.7 \cdot 10^{-8}, \quad k_{conv} = 1.34, \quad \rho_{20} = 1.7241 \cdot 10^{-8}, \quad \alpha_T = 0.00393 \]
\[ T_a = 293.15, \quad k_{rad} = 5.700000000 \cdot 10^{-8}, \quad k_{conv} = 1.34, \quad \rho_{20} = 1.724100000 \cdot 10^{-8}, \quad \alpha_T = 0.00393 \]  
\[ (1) \]

\[ \text{MatlCons} := k_{hys} = 50.87, \quad x = 1.6, \quad k_{eddy} = 5.89 \cdot 10^{6}, \quad t = 0.35 \cdot 10^{-3}, \quad B_{Max} = 1.5, \quad \epsilon = 0.9 \]
\[ \quad k_{hys} = 50.87, \quad x = 1.6, \quad k_{eddy} = 5.890000000 \cdot 10^{6}, \quad t = 0.0003500000000, \quad B_{Max} = 1.5, \quad \epsilon = 0.9 \]  
\[ (2) \]

\[ \text{XfmrCons} := S_r = 10 \cdot 10^3, f = 60, \quad E_p = 480, \quad E_s = 220, \quad k_u = 0.4, \quad T_s = 273.15 + 45 \]
\[ \quad S_r = 10000, f = 60, \quad E_p = 480, \quad E_s = 220, \quad k_u = 0.4, \quad T_s = 318.15 \]  
\[ (3) \]

\[ \text{CoreCons} := k_s = 39.2, \quad k_v = 17.9 \]
\[ \quad k_s = 39.2, \quad k_v = 17.9 \]  
\[ (4) \]

\[ \text{params} := \{ \text{Cons, MatlCons, XfmrCons, CoreCons} \} \]
\[ \{ f = 60, \quad t = 0.0003500000000, \quad x = 1.6, \quad \epsilon = 0.9, \quad B_{Max} = 1.5, \quad E_p = 480, \quad E_s = 220, \quad S_r = 10000, \quad T_a = 293.15, \quad T_s = 318.15, \quad k_{hys} = 50.87, \quad k_{rad} = 5.700000000 \cdot 10^{-8}, \quad k_s = 39.2, \quad k_u = 0.4, \quad k_v = 17.9, \quad k_{conv} = 1.34, \quad k_{eddy} = 5.890000000 \cdot 10^{6}, \quad \rho_{20} = 1.724100000 \cdot 10^{-8}, \quad \alpha_T = 0.00393 \} \]  
\[ (5) \]

\[ \text{Eq}(7.22) \]
\[ P_{rad} := k_{rad} \cdot \epsilon \cdot k_s \cdot A_p^{\frac{1}{2}} \cdot (T_s^4 - T_a^4) \]
\[ k_{rad} \cdot \epsilon \cdot k_s \cdot \sqrt{A_p} \cdot (T_s^4 - T_a^4) \]  
\[ (6) \]

\[ \text{Eq}(7.23) \]
\[ P_{conv} := k_{conv} \cdot (T_s - T_a)^{\frac{5}{4}} \cdot k_s \cdot k_v \cdot \frac{1}{12} \cdot A_p^{\frac{7}{16}} \]
\[ k_{conv} \cdot (T_s - T_a)^{\frac{5}{4}} \cdot k_s \cdot k_v \cdot \frac{1}{12} \cdot A_p^{\frac{7}{16}} \]  
\[ (7) \]

\[ \text{Eq}(7.44) \]
\[ P_{cu} := k_u \cdot J^2 \cdot \rho_{20} \left( 1 + \frac{T_s - 293.15}{\frac{1}{\alpha_T} + (T_s - 293.15)} \right) \cdot \left( \frac{1}{4 \cdot k^2} + 2 \cdot k^2 \right) \cdot A_p^{\frac{3}{4}} \]
\[ k_u \cdot J^2 \cdot \rho_{20} \left( 1 + \frac{T_s - 293.15}{\frac{1}{\alpha_T} + (T_s - 293.15)} \right) \cdot \left( 4 \sqrt{k} + 2 \cdot k^{3/2} \right) \cdot A_p^{\frac{3}{4}} \]  
\[ (8) \]
Eq (7.50)

\[
P_{fe} := \left( k_{\text{hys}} f B_{\text{Max}}^2 + k_{\text{eddy}} f^2 \sqrt{2} B_{\text{Max}}^2 \right) \left( 4 \cdot k^{-1/2} + 4 \cdot k^{-3/2} \right) \cdot A_p^{3/4}
\]
\[
\left( k_{\text{hys}} f B_{\text{Max}}^x + k_{\text{eddy}} f^2 \sqrt{2} B_{\text{Max}}^2 \right) \left( \frac{4}{\sqrt{k}} + \frac{4}{k^{3/2}} \right) A_p^{3/4}
\]  

(9)

Eq (7.21) (This eliminates \(A_p\) from the following two equations.)

\[
A_p := \frac{\sqrt{2} \cdot S_r}{2 \cdot \pi f B_{\text{Max}} \cdot J \cdot k_u}
\]
\[
\frac{1}{2} \frac{\sqrt{2} \cdot S_r}{\pi f B_{\text{Max}} J k_u}
\]  

(10)

First

\[
P_{\text{rad}} + P_{\text{conv}} = P_{\text{cu}} + P_{fe}
\]
\[
\frac{1}{2} k_{\text{rad}} e k_s \sqrt{2} \sqrt{\frac{\sqrt{2} \cdot S_r}{2 \cdot \pi f B_{\text{Max}} J k_u}} \left( T_s - T_a \right)
\]
\[
+ \frac{k_{\text{conv}} (T_s - T_a)^{5/4} k_s 2^{9/16}}{k_u^{(1/12)}} \left( \frac{\sqrt{2} \cdot S_r}{\pi f B_{\text{Max}} J k_u} \right)^{7/16} = \frac{1}{2} k_u J^2 \rho_{20} \left\{ 1
\right.
\]
\[
+ \frac{T_s - 293.15}{1 + T_s - 293.15} \left( 4 \sqrt{k} + 2 k^{3/2} \right) 2^{1/4} \left( \frac{\sqrt{2} \cdot S_r}{\pi f B_{\text{Max}} J k_u} \right)^{3/4}
\]
\[
+ k_{\text{eddy}} f^2 \sqrt{2} B_{\text{Max}}^2 \left( \frac{4}{\sqrt{k}} + \frac{4}{k^{3/2}} \right) 2^{1/4} \left( \frac{\sqrt{2} \cdot S_r}{\pi f B_{\text{Max}} J k_u} \right)^{3/4}
\]

Next

\[
P_{\text{cu}} = P_{fe}
\]
\[
\frac{1}{2} k_u J^2 \rho_{20} \left\{ 1 + \frac{T_s - 293.15}{1 + T_s - 293.15} \left( 4 \sqrt{k} + 2 k^{3/2} \right) 2^{1/4} \left( \frac{\sqrt{2} \cdot S_r}{\pi f B_{\text{Max}} J k_u} \right)^{3/4}
\right.
\]
\[
+ k_{\text{eddy}} f^2 \sqrt{2} B_{\text{Max}}^2 \left( \frac{4}{\sqrt{k}} + \frac{4}{k^{3/2}} \right) 2^{1/4} \left( \frac{\sqrt{2} \cdot S_r}{\pi f B_{\text{Max}} J k_u} \right)^{3/4}
\]

(12)
\[ \text{eval((11), params)} \]
\[
47931.51384 \sqrt{2} \frac{\sqrt{2}}{\pi J} + 13536.27338 \, 2^{9/16} \left( \frac{\sqrt{2}}{\pi J} \right)^{7/16} = 2.556095424 \times 10^{-7} \, J^2 \left( 4 \frac{1}{\sqrt{k}} + \frac{4}{k^{3/2}} \right) k^{3/2} 2^{1/4} \left( \frac{\sqrt{2}}{\pi J} \right)^{3/4} \]

\[ \text{eval((12), params)} \]
\[
2.556095424 \times 10^{-7} \, J^2 \left( 4 \frac{1}{\sqrt{k}} + 2 \frac{4}{k^{3/2}} \right) k^{3/2} 2^{1/4} \left( \frac{\sqrt{2}}{\pi J} \right)^{3/4} = 3.974851132 \times 10^5 \left( \frac{4}{\sqrt{k}} \right) \frac{4}{k^{3/2}} 2^{1/4} \left( \frac{\sqrt{2}}{\pi J} \right)^{3/4} \]

Initial Conditions

The maximum current density of copper is usually taken as \( 4 \frac{A}{mm^2} = 4 \times 10^6 \frac{A}{m} \). If we take \( \frac{1}{10} \) of the maximum, our estimate for \( J = \frac{4 \times 10^6}{10} \). Based on observations concerning the size of 10 kVA transformers, we let our initial estimate of \( k \) be \( k = 2 \).

\[ \text{fsolve}\left( \{(13), (14)\}, \left\{ k = 2, J = \frac{4 \times 10^6}{10} \right\} \right) \]
\[ \left\{ J = 8.247347431 \times 10^5, k = 1.838835931 \right\} \]

\[ \text{eval((10), params)} \]
\[
\frac{138.8888889 \sqrt{2}}{\pi J} \]

Eq(7.21)
\[ A_p = \text{evalf(eval((16), (15)))) = A_p = 0.00007580857591} \]

Eq(7.37) \[ a = \frac{A_p}{k^2} \]
\[
a = \frac{0.00007580857591}{1.838835931^{1/2}} = a = 0.06881103560 \]
Once we have solved for \( J \) and \( k \) as shown on page 7-17, we can easily solve for \( A_p \) and \( a \), as shown on page 7-18. We now need to specify the number of primary and secondary turns, and the diameter of the wire to be used for the windings. We begin by considering the ratio of the window area to the core area \( \frac{A_W}{A_c} \). Solving (7.20) for \( A_W \) we have

\[
A_W = \frac{A_p}{A_c}
\]  
(7.51)

Substitution of (7.51) into the ratio \( \frac{A_W}{A_c} \) gives

\[
\frac{A_W}{A_c} = \left( \frac{A_p}{A_c} \right) = \frac{A_p}{A_c^2}
\]  
(7.52)

On the other hand, substitution of (7.37) and (7.38) into the ratio gives

\[
\frac{A_W}{A_c} = \frac{k^2 a^2}{a^2} = k^2
\]  
(7.53)

If we solve (7.8) for \( A_c \) we obtain

\[
A_c = \frac{\sqrt{2} E_p}{n_p \omega B_{Max}}
\]  
(7.54)

Substitution of (7.54) and (7.53) into (7.52) gives

\[
\frac{A_W}{A_c} = \frac{A_p}{A_c^2} = k^2 = \frac{A_p}{\left( \frac{\sqrt{2} E_p}{n_p \omega B_{Max}} \right)^2}
\]  
(7.55)

Solving the rightmost equality in (7.55) for \( n_p \) produces

7-19
\[ n_p = \frac{\sqrt{2} k E_p}{\sqrt{A_p} \omega B_{Max}} \]  
(7.56)

Which is easily solved for the number of primary turns \( n_p \). The number of secondary turns \( n_s \) is found as \( n_s = \frac{n_p}{n} \) where \( n \) is found by solving (7.1) as \( n = \frac{E_p}{E_s} \) which gives

\[ n_s = \left( \frac{E_p}{E_s} \right) \frac{n_p}{n} \]  
(7.57)

The diameter of the wires (the primary and secondary wire diameters are different) that yield the correct current density \( J \) can now be easily found via (7.13) as

\[ \text{Dia}_{pri} = \sqrt{\frac{4 CA_p}{\pi}} = \sqrt{\frac{2 K_u A_W}{\pi n_p}} = \sqrt{\frac{2 K_u k^2 a^2}{\pi n_p}} \]  
(7.58)

and

\[ \text{Dia}_{sec} = \sqrt{\frac{4 CA_s}{\pi}} = \sqrt{\frac{2 K_u A_W}{\pi n_s}} = \sqrt{\frac{2 K_u k^2 a^2}{\pi n_s}} \]  
(7.59)

The design is now complete, except for the selection of standard wire sizes for the conductors. Before we investigate the American Standard Wire Gauge sizes, let us summarize the method.

Given \( f, S_r, E_p, E_s, B_{Max}, k_h, k_t, k_{s}, x, k_r, t, T_r, \) and \( T_s \), find \( k, I, A_p, \) and \( a \) via the equations shown in the Maple solution, then use (7.56) and (7.57) to obtain \( n_p \) and \( n_s \) and finally determine \( \text{Dia}_{pri} \) and \( \text{Dia}_{sec} \) from (7.58) and (7.59). The only remaining step is to choose a standard wire size for the primary winding and for the secondary winding.

For our example the above process yields:

\[ n_p = 254 \]
\[ n_s = 117 \]
\[ \text{Dia}_{pri} = 4.01 \text{ mm} \]
\[ \text{Dia}_{sec} = 5.90 \text{ mm} \]

The calculations are shown on pages 7-21 and 7-22.
\[
Cons := T_a = 273.15 + 20, \ k_{\text{rad}} = 5.7 \cdot 10^{-8}, \ k_{\text{conv}} = 1.34, \ \rho_{20} = 1.7241 \cdot 10^{-8}, \ \alpha_T = 0.00393
\]
\[
T_a = 293.15, \ k_{\text{rad}} = 5.700000000 \cdot 10^{-8}, \ k_{\text{conv}} = 1.34, \ \rho_{20} = 1.724100000 \cdot 10^{-8}, \ \alpha_T = 0.00393
\] (1)

\[
MailCons := k_{\text{hys}} = 50.87, x = 1.6, \ k_{\text{eddy}} = 5.89 \cdot 10^{6}, \ t = 0.35 \cdot 10^{-3}, \ B_{\text{Max}} = 1.5, \ \varepsilon = 0.9
\]
\[
k_{\text{hys}} = 50.87, x = 1.6, \ k_{\text{eddy}} = 5.890000000 \cdot 10^{6}, \ t = 0.0003500000000, \ B_{\text{Max}} = 1.5, \ \varepsilon = 0.9
\] (2)

\[
XfmrCons := S_r = 10 \cdot 10^3, f = 60, \ E_p = 480, \ E_s = 220, \ k_u = 0.4, \ T_s = 273.15 + 45
\]
\[
S_r = 10000, f = 60, \ E_p = 480, \ E_s = 220, \ k_u = 0.4, \ T_s = 318.15
\] (3)

\[
CoreCons := k_s = 39.2, k_v = 17.9
\]
\[
k_s = 39.2, k_v = 17.9
\] (4)

\[
params := \{Cons, MailCons, XfmrCons, CoreCons\}
\]
\[
\{f = 60, t = 0.0003500000000, x = 1.6, \varepsilon = 0.9, B_{\text{Max}} = 1.5, E_p = 480, E_s = 220, S_r = 10000, T_a
\]
\[
= 293.15, T_s = 318.15, k_{\text{hys}} = 50.87, k_{\text{rad}} = 5.700000000 \cdot 10^{-8}, k_s = 39.2, k_u = 0.4, k_v = 17.9, k_{\text{conv}}
\]
\[
= 1.34, k_{\text{eddy}} = 5.890000000 \cdot 10^{6}, \rho_{20} = 1.724100000 \cdot 10^{-8}, \alpha_T = 0.00393\}
\] (5)

\[
results := \{J = 8.247347431 \cdot 10^5, k = 1.838835931, A_p = 0.00007580857591, a = 0.06881103560\}
\]
\[
\{J = 8.247347431 \cdot 10^5, a = 0.06881103560, k = 1.838835931, A_p = 0.00007580857591\}
\] (6)

\[
n_p = \text{ceil} \left( \frac{\sqrt{2} \cdot k \cdot E_p}{\sqrt{A_p \cdot 2 \cdot \pi \cdot f \cdot B_{\text{Max}}}} \right)
\]
\[
n_p = \text{ceil} \left( \frac{1}{2} \cdot \frac{\sqrt{2} \cdot k \cdot E_p}{\sqrt{A_p \cdot \pi f \cdot B_{\text{Max}}}} \right)
\] (7)

\[
eval(\text{eval((7), params union results)})
\]
\[
n_p = 254.
\] (8)

\[
n_s = \frac{n_p}{\left(\frac{E_p}{E_s}\right)}
\]
\[
n_s = \frac{n_p \cdot E_s}{E_p}
\] (9)

\[
eval((9), \text{params union (8)})
\]
\[
n_s = 116.4166667
\]
\[
7-21
\] (10)
\[ n_s = \text{ceil}(rhs((10))) \]

\[ n_s = 117 \]  \hspace{1cm} (11)

\[ \text{Dia}_{pri} = \sqrt{\frac{2 \cdot k_u \cdot a^2 \cdot k^2}{\pi \cdot n_p}} \]

\[ \text{Dia}_{pri} = \sqrt{2} \sqrt{\frac{k_u \cdot a^2 \cdot k^2}{\pi \cdot n_p}} \]  \hspace{1cm} (12)

\[ \text{eval}(\text{eval}((12), \text{params union results union } ((8)))) \]

\[ \text{Dia}_{pri} = 0.004006399693 \]  \hspace{1cm} (13)

\[ \text{Dia}_{sec} = \sqrt{\frac{2 \cdot k_u \cdot a^2 \cdot k^2}{\pi \cdot n_s}} \]

\[ \text{Dia}_{sec} = \sqrt{2} \sqrt{\frac{k_u \cdot a^2 \cdot k^2}{\pi \cdot n_s}} \]  \hspace{1cm} (14)

\[ \text{eval}(\text{eval}((14), \text{params union results union } ((11)))) \]

\[ \text{Dia}_{sec} = 0.005903073645 \]  \hspace{1cm} (15)

7-22
AMERICAN WIRE GAUGE
The American wire gauge assigns numbers to wires in standard sizes. The numbers run from 
#0000 (written 4/0 and pronounced “four-aught”) to #46 in the following sequence:

<table>
<thead>
<tr>
<th>Wire GAUGE</th>
<th>Diam (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#0000</td>
<td>4/0</td>
</tr>
<tr>
<td>#000</td>
<td>3/0</td>
</tr>
<tr>
<td>#00</td>
<td>2/0</td>
</tr>
<tr>
<td>#0</td>
<td>1/0</td>
</tr>
<tr>
<td>#1</td>
<td></td>
</tr>
<tr>
<td>#2</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
<tr>
<td>#35</td>
<td></td>
</tr>
<tr>
<td>#36</td>
<td></td>
</tr>
<tr>
<td>#37</td>
<td></td>
</tr>
<tr>
<td>#46</td>
<td></td>
</tr>
</tbody>
</table>

The standard is created by defining the diameter of #4/0 as 0.4600 inches, and the diameter of 
#36 as 0.005 inches. The diameters of the intervening sizes are chosen to be evenly spaced 
between #4/0 and #36 as follows: each size has a diameter that is a constant multiplied by the 
diameter of the next largest size. So the diameter of #000 = k(0.46) in., the diameter of #00 is 
given as k(k(0.46)) = k²(0.46) in., the diameter of #0 = k(k²(0.46)) = k³(0.46) in., and so on. The 
diameter of #36 = k³⁹(0.46) = 0.005 in., so we may write:

\[ 0.005 = k^{39} \cdot 0.4600 \]  

which is easily solved for \( k \) as

\[ k = \left( \frac{0.005}{0.4600} \right)^{\frac{1}{39}} = \left( \frac{1}{92} \right)^{\frac{1}{39}} = 0.890526 \]  

The diameter of \(#N\) is then given as

\[ \text{Dia}_{N} = (0.4600) \cdot k^{N + 3} = \left( \frac{1}{92} \right)^{\frac{N}{39}} \cdot (0.4600) \]  

where \( N = 0 \) for 1/0, \( N = -1 \) for 2/0, \( N = -2 \) for 3/0, and \( N = -3 \) for 4/0. Since we have the 
diameter of the primary winding from (7.58), we may use (7.62) to write an equation for \( N \) given 
\( \text{Dia}_{pri} \):

\[ \left( \frac{\text{Dia}_{pri} \ (m)}{0.4600 \ (in)} \right) = \left( \frac{1}{92} \right)^{\frac{N + 3}{39}} \]  

7-23
Eq. (7.63) is solved for \( N \) by taking the natural log of both sides, which yields

\[
\ln \left( \frac{39.37}{0.46} \cdot \text{Dia}_{pri} \right) = \ln \left( \left( \frac{1}{92} \right)^{\frac{N+3}{39}} \right) = \left( \frac{N + 3}{39} \right) \ln \left( \frac{1}{92} \right) \quad (7.64)
\]

\( N \) can be isolated from (7.64) as

\[
N = 39 \left( \frac{\ln \left( 85.59 \cdot \text{Dia}_{pri} \right)}{\ln \left( \frac{1}{92} \right)} \right)^{-3} = -8.625 \cdot \ln \left( 85.59 \cdot \text{Dia}_{pri} \right)^{-3} \quad (7.65)
\]

Since in our example we have \( \text{Dia}_{pri} = 0.00402 \), Eq. (7.65) gives \( N = \#6.2 \) as the required wire size. Of course, \#6.2 is not a standard size so we must use the next largest size \#6. A similar calculation gives the size of the secondary conductors as \#2. The use of (7.65) can be avoided by the use of a standard wire size table that can be found in any electrical engineering handbook. One determines the required wire diameter, and then looks in the wire table for a wire that has the necessary diameter. An example of such a table (this one was done in Maple) is shown on page 7-26.

When one consults the wire size tables in most handbooks, one finds sizes larger than 4/0 given in kcmil or kilo-circular mils. A circular mil is defined as the area of a circle whose diameter is one mil, where one mil is one thousandth of an inch (one milli-inch).

\[
1 \text{ cmil} = \pi \left( \frac{1}{2} \text{ mil} \right)^2 = 0.7854 \text{ mil}^2 = 7.854 \cdot 10^{-7} \text{ in}^2 \quad (7.66)
\]

A wire with diameter \( d \) mils has area \( d^2 \) cmils or \( \frac{d^2}{1000} \) kcmils.

A wire with area \( c \) cmil has a diameter of \( \sqrt{c} \) mils e.g. a 100 cmil wire has a diameter of 10 mils.

A wire with area \( b \) kcmil has a diameter of \( \sqrt{b \cdot 1000} \) mils e.g. a 250 kcmil wire has a diameter of 500 mils.

A table of standard wire sizes in kcmils and their diameters is given on page 7-27.

7-24
American Wire Gauge

\textit{interface (rtablesize = 25)}

\begin{equation}
AWG := \textit{Matrix}(1..23, 1..7):
\end{equation}

\begin{align*}
n1 & := \textit{seq}(i - 3, i = 0..22) \\
& \quad \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19\} \\
n2 & := \textit{seq}(i - 3, i = 23..44) \\
& \quad \{20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41\}
\end{align*}

\begin{align*}
& \quad \{20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41\}
\end{align*}

\begin{align*}
AWG_{k, 1} & := n:\nAWG_{k, 2} & := \textit{Dia}_{in}:\nAWG_{k, 3} & := \textit{Dia}_{mn}:
\end{align*}

\begin{algorithm}
\begin{algorithmic}
\FOR {k from 2 to 23}
\STATE $AWG_{k, 1} := n_{k-1}$
\STATE $AWG_{k, 2} := \textit{evalf}\left(0.46 \cdot \left(\frac{\text{AWG}_{k, 1}}{39} + 3\right)\right)$
\STATE $AWG_{k, 3} := \frac{\text{AWG}_{k, 2} \cdot 1000}{39.37}$
\ENDFOR
\end{algorithmic}
\end{algorithm}

\begin{algorithm}
\begin{algorithmic}
\FOR {k from 2 to 23}
\STATE $AWG_{k, 4} := \cdot \cdot \cdot$
\STATE $AWG_{k, 5} := n$
\STATE $AWG_{k, 6} := \textit{Dia}_{in}$
\STATE $AWG_{k, 7} := \textit{Dia}_{mn}$
\ENDFOR
\end{algorithmic}
\end{algorithm}

\begin{algorithm}
\begin{algorithmic}
\FOR {k from 2 to 23}
\STATE $AWG_{k, 8} := n_{k-1}$
\STATE $AWG_{k, 9} := \textit{evalf}\left(0.46 \cdot \left(\frac{\text{AWG}_{k, 5}}{39} + 3\right)\right)$
\STATE $AWG_{k, 10} := \frac{\text{AWG}_{k, 9} \cdot 1000}{39.37}$
\ENDFOR
\end{algorithmic}
\end{algorithm}

\begin{algorithm}
\begin{algorithmic}
\FOR {k from 2 to 23}
\STATE $AWG_{k, 11} := \cdot \cdot \cdot$
\STATE $AWG_{k, 12} := \cdot \cdot \cdot$
\ENDFOR
\end{algorithmic}
\end{algorithm}
\begin{align*}
\begin{array}{|c|c|c|c|}
\hline
n & \mathit{Dia_{in}} & \mathit{Dia_{mm}} & n & \mathit{Dia_{in}} & \mathit{Dia_{mm}} \\
\hline
-3 & 0.46 & 11.68402337 & 20 & 0.03196145552 & 0.8118225938 \\
-2 & 0.4096418300 & 10.40492329 & 21 & 0.02846249812 & 0.7229488981 \\
-1 & 0.3647965846 & 9.265851781 & 22 & 0.02534658656 & 0.6438045862 \\
0 & 0.3248607402 & 8.251479304 & 23 & 0.02257178718 & 0.5733245110 \\
1 & 0.2892968438 & 7.348154529 & 24 & 0.02010075697 & 0.5105602482 \\
2 & 0.2576262794 & 6.543720584 & 25 & 0.01790024102 & 0.4546670312 \\
3 & 0.2294228273 & 5.827351468 & 26 & 0.01594062498 & 0.4048926843 \\
4 & 0.2043069278 & 5.189406345 & 27 & 0.01419553650 & 0.3605673482 \\
5 & 0.1819405736 & 4.621299812 & 28 & 0.01264149032 & 0.3210944963 \\
6 & 0.1620227598 & 4.115386330 & 29 & 0.01125757224 & 0.2859429068 \\
7 & 0.1442854344 & 3.664857363 & 30 & 0.01002515760 & 0.2546395123 \\
8 & 0.1284898900 & 3.263649733 & 31 & 0.008927660660 & 0.2267630343 \\
9 & 0.1144235515 & 2.906364021 & 32 & 0.007950311415 & 0.2019383138 \\
10 & 0.1018971153 & 2.588191905 & 33 & 0.007079956780 & 0.1798312619 \\
11 & 0.09074200170 & 2.304851453 & 34 & 0.006304883590 & 0.1601443635 \\
12 & 0.08080808615 & 2.052529493 & 35 & 0.005614660985 & 0.1426127642 \\
13 & 0.07196167890 & 1.827830300 & 36 & 0.005000000000 & 0.1270002540 \\
14 & 0.06408372575 & 1.627729890 & 37 & 0.004452628588 & 0.1130969923 \\
15 & 0.05706820585 & 1.449535328 & 38 & 0.003965180268 & 0.1007157802 \\
16 & 0.05082070495 & 1.290848487 & 39 & 0.003531095003 & 0.0896899246 \\
17 & 0.04525714474 & 1.149533775 & 40 & 0.003144530911 & 0.0798712448 \\
18 & 0.04030265130 & 1.023689390 & 41 & 0.002800285646 & 0.0711273976 \\
\hline
\end{array}
\end{align*}

\begin{align}
K_{cmil} & := \text{Matrix}(1..16, 1..4) : \\
K_{cmil,1} & := \text{cmil} : \\
K_{cmil,2} & := \text{Dia}_{in} : \\
K_{cmil,3} & := \text{Dia}_{mm} : \\
K_{cmil,4} & := \text{Dia}_{mm} : \\
n3 & := [250, 300, 350, 400, 500, 600, 700, 750, 800, 900, 1000, 1250, 1500, 1750, 2000] \\
& \quad [250, 300, 350, 400, 500, 600, 700, 750, 800, 900, 1000, 1250, 1500, 1750, 2000] \\
\textbf{for} \ k \ \textbf{from} \ 2 \ \textbf{to} \ 16 \ \textbf{do} \\
K_{cmil,k,1} & := n3_{k-1} : \\
K_{cmil,k,2} & := \text{evalf}\left(\frac{\sqrt{K_{cmil,k,1} \cdot 1000}}{1000}\right) : \\
K_{cmil,k,3} & := \text{evalf}\left(\frac{\sqrt{K_{cmil,k,1} \cdot 1000}}{1000}\right) : \\
K_{cmil,k,4} & := \text{evalf}\left(\frac{\sqrt{K_{cmil,k,1} \cdot 1000}}{1000}\right) \cdot 25.4 : \\
\textbf{end do}:
\end{align}

7-26
<table>
<thead>
<tr>
<th>kmil</th>
<th>( D_{in} )</th>
<th>( D_{mils} )</th>
<th>( D_{nm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>0.5000000000</td>
<td>500.</td>
<td>12.70000000</td>
</tr>
<tr>
<td>300</td>
<td>0.5477225575</td>
<td>547.7225575</td>
<td>13.91215296</td>
</tr>
<tr>
<td>350</td>
<td>0.5916079783</td>
<td>591.6079783</td>
<td>15.02684265</td>
</tr>
<tr>
<td>400</td>
<td>0.6324555320</td>
<td>632.4555320</td>
<td>16.06437051</td>
</tr>
<tr>
<td>500</td>
<td>0.7071067810</td>
<td>707.1067810</td>
<td>17.96051224</td>
</tr>
<tr>
<td>600</td>
<td>0.7745966692</td>
<td>774.5966692</td>
<td>19.67475540</td>
</tr>
<tr>
<td>700</td>
<td>0.8366000265</td>
<td>836.6600265</td>
<td>21.25116467</td>
</tr>
<tr>
<td>750</td>
<td>0.8660254040</td>
<td>866.0254040</td>
<td>21.99704526</td>
</tr>
<tr>
<td>800</td>
<td>0.8944271908</td>
<td>894.4271908</td>
<td>22.71845065</td>
</tr>
<tr>
<td>900</td>
<td>0.9486832980</td>
<td>948.6832980</td>
<td>24.09655577</td>
</tr>
<tr>
<td>1000</td>
<td>1.0000000000</td>
<td>1000.</td>
<td>25.4</td>
</tr>
<tr>
<td>1250</td>
<td>1.118033988</td>
<td>1118.033988</td>
<td>28.39806330</td>
</tr>
<tr>
<td>1500</td>
<td>1.224744872</td>
<td>1224.744872</td>
<td>31.10851975</td>
</tr>
<tr>
<td>1750</td>
<td>1.322875656</td>
<td>1322.875656</td>
<td>33.60104166</td>
</tr>
<tr>
<td>2000</td>
<td>1.414213562</td>
<td>1414.213562</td>
<td>35.92102447</td>
</tr>
</tbody>
</table>

(7)
APPENDIX I OTHER CORE STYLES

"E I" CORES
In addition to the "C" core there are several other popular configurations, each of which has its own values for \( k_e \) and \( k_w \). The first core we examine is closely related to the "C" core, and its name "E I" core is also based on resemblance to letters of the alphabet. A stylized drawing of an "E I" core transformer is shown in Fig. AI-1 below. (The top portion forms the "I" and the lower portion forms the "E".)

![Diagram](https://via.placeholder.com/150)

**Fig. AI-1** "E I" Core Transformer

In Fig. AI-1 only the primary winding is shown; the secondary winding is wound over the primary winding. In some transformers bifilar windings are used to improve the coefficient of coupling, and to improve the symmetry of their windings. Bifilar windings are produced by winding one layer of the primary, then winding the first layer of the secondary over the first layer of the primary. Next comes the second layer of the primary, then the second layer of the secondary, and so on. Notice that the dimensions of the "E I" core are chosen so that \( A_w \) is still \((ka)^2\) (Since each turn must pass through both windows, we only count the area of one window as \( A_w \)), and that \( A_e \) is still \( a^2 \). (The flux splits into two paths: one to the right and one to the left.)
The core coefficients are $k_s = 41.3$ and $k_v = 19.7$ for the "E I" core. A fully wound "E I" core transformer is shown in Fig. AI-2 below.

![Diagram of a core transformer](image)

$$A_S = 2 \left[ (A + 2B)(A + 2B + 2C) - 4BC + (E + 2D)(A + 2B) - 4BD + (E + 2D)(A + 2B + 2C) \right]$$

Fig. AI-2 Fully wound "E I" Core Transformer

The mean turn length of an "E I" transformer can be determined from Fig. AI-3 as

$$MTL = 2 \cdot \left( \frac{2a}{\sqrt{2}} + 2 \left( \frac{k a}{2} \right) + \left( \frac{a}{\sqrt{2}} + 2 \left( \frac{k a}{2} \right) \right) \right) = \frac{6a}{\sqrt{2}} + 4ka \quad (AI-1)$$

![Diagram of top view of core transformer](image)

Fig. AI-3 Top View of "E I" Core Transformer
All the equations involving the MTL must, of course, be modified according to (AI-1).

There are two other core styles that are commonly used in the design of transformers: the pot core and the torroid core. These cores find widespread use in the design of transformers for switchmode power supplies (see Appendix III).

POT CORES
The pot core is so named because of its resemblance to a cooking pot with no handle. The “pot” has a center post around which the windings are placed, one over the other in the same manner as in an “EI” core transformer. The flux path is up through the center post, radially through the lid, down the sides of the “pot” and radially through the bottom back to the center post. Pot core transformers are well shielded, since the core surrounds the windings completely. A pot core is illustrated in Fig. AI-4, below, where \( a \) is the radius of the center post, \( k \alpha \) is the distance between the edge of the center post and the edge of the outer wall, \( t_w \) is the thickness of the outer wall, and \( t_L \) is the thickness of the lid. \( A_{\text{Core}} = \pi a^2 \) and \( A_{\text{Window}} = (k \alpha)^2 \) (The height is restricted to \( k \alpha \) to simplify the design.)
In order to ensure constant flux throughout the device, we make the following restrictions. First we set the cross-sectional area of the entire outer wall equal to the cross-sectional area of the center post (All of the flux which flows in the center post is distributed in the outer wall.) which gives

$$\pi a^2 = \pi (a + k a + t_w)^2 - \pi (a + ka)^2$$

(AI-2)

Cancelling $\pi$ from both sides of (AI-2) and expanding the squared terms we have

$$a^2 + 2ka^2 + k^2a^2 + 2t_w(a + ka) + t_w^2 - (a^2 + 2ka^2 + k^2a^2) = a^2$$

(AI-3)

After some cancellation we have

$$t_w^2 + 2(a + ka) t_w - a^2 = 0$$

(AI-4)

Solving for $t_w$ by the quadratic formula, we have

$$t_w = \frac{-2(a + ka)}{2} \pm \frac{\sqrt{4(a + ka)^2 + 4a^2}}{2}$$

(AI-5)

Since $t_w$ must be positive, we take the plus sign in (AI-5), and after some simplification we have

$$t_w = a \sqrt{2 + 2k + k^2} - a(k + 1)$$

(AI-6)

Since all of the flux leaving the center post to enter the lid must pass through a cylinder whose area is $2 \pi a t_L$, we require the area of the cylinder and the area of the center post to be the same so

$$2 \pi a t_L = \pi a^2$$

(AI-7)

Solving (AI-7) for $t_L$ yields

$$t_L = \frac{a}{2}$$

(AI-8)

The core parameters for pot cores are: $k_z = 33.8$ $k_r = 14.5$

As can be seen from Fig. AI-4, the mean turn length (MTL) is

$$MTL = 2 \pi \left( a + \frac{ka}{2} \right)$$

(AI-9)
TOROIDAL CORES

The toroidal (or “tape-wound toroid”) core is a toroid with a square cross-section as shown in Fig. AI-5 below. (Only a few turns are shown to avoid cluttering the drawing; the windings continue around the circumference of the toroid in several layers.)

![Toroidal Core Diagram]

**Front View**  
**Side View**  
(Cross-Section)

**Fig. AI - 5**  
Toroidal Core

For toroidal transformers $A_c = a^2$, and $A_w = \pi (ka)^2$. The main advantage of the toroidal transformer is that it uses the flux most efficiently since the leakage is very low. The main disadvantage is that the winding may occupy only a small portion of the window, since the spool of wire from which the windings are placed on the transformer must be able to pass through the window, even on the final turns of the transformer. (The windings are placed one over the other as in “E I” core and pot core transformers.) This means that a significant empty space must exist in the center of the transformer, and the window utilization factor $k_u$ for this type of transformer is not $k_u = 0.4$ like all the rest of the transformers, but $k_u = 0.25$, which is significantly smaller. The mean turn length $MTL = (4a + 2ka)$. (See Fig. AI-5)

The core parameters for toroidal cores are $k_s = 50.9$  $k_s = 25.0$
APPENDIX II  EXTRACTION OF $k_h$ AND $k_e$ FROM HANDBOOK TABLES

Manufacturers of electrical grade steel (high permeability steel for use in transformer cores and rotating electric machines) publish tables of characteristics of these materials for use in selecting core material. Most electrical engineering handbooks (see [3] for instance) contain reprints of these tables to aid engineers in the selection of core material. The final decision as to the particular core to use is essentially an economic one: materials with better magnetic performance are more expensive. In the example problem on page 7-15 we gave values for $B_{\text{Max}}$, $k_h$, $x$, $k_e$, and t. All the values except $k_h$ and $k_e$ were taken directly from [3]. However, it is often necessary to calculate $k_h$ and $k_e$ from the information that is given in the table. What is given in these table is the maximum loss in Watts per pound. If we examine (7.47) on page 7-14 we see that if we divide both sides by the core volume, the left hand side is the loss in Watts per cubic meter, and the right hand side consists of known parameters and the two unknowns $k_h$ and $k_e$. Now the loss in Watts per pound can be converted to the loss in Watts per cubic meter in the following way. Iron has a density of approximately 0.29 pounds per cubic inch, and there are approximately 39.37 inches per meter. So to calculate the loss in Watts per cubic meter, we multiply the loss in Watts per pound by 0.29 pounds per cubic inch, then multiply by $(39.37)^3$ cubic inches per cubic meter. The extraneous units cancel as in (AII-1) leaving the loss in Watts per cubic meter.

$$\text{Loss} \left( \frac{W}{m^3} \right) = \text{Loss} \left( \frac{W}{lb} \right) \cdot 0.29 \frac{lb}{in^3} \cdot (39.37)^3 \frac{in^3}{m^3} \quad (\text{AII-1})$$

Or

$$\text{Loss} \left( \frac{W}{m^3} \right) = \text{Loss} \left( \frac{W}{lb} \right) \cdot 17696.8 \frac{lb}{m^3} \quad (\text{AII-2})$$

In our example [3] gives the loss for M6 grain oriented steel as 0.66 W/lb @ B = 1.5 T. So the loss in W/m³ is given by (AII-2) as 11679.89 and substitution for the other terms on the right hand side of (7.47) gives

$$11679.89 = 114.8 \, k_h + 9.923 \times 10^{-4} \, k_e \quad (\text{AII-3})$$

We require a second equation for a solveable system, and a common assumption is that the losses are split about evenly between the hysteresis losses and the eddy current losses. If we make the above assumption (AII-3) gives:

$$11679.89 \left( \frac{1}{2} \right) = 114.8 \, k_h \rightarrow k_h = 50.87 \quad (\text{AII-4})$$

and

$$11679.89 \left( \frac{1}{2} \right) = 9.923 \times 10^{-4} \, k_e \rightarrow k_e = 5.89 \times 10^6 \quad (\text{AII-5})$$

When the material is available for testing, a relatively simple test can be used to determine how the total core loss is split between the hysteresis loss and the eddy current loss. The core losses
are determined (i.e. the open circuit test is run) at two different frequencies \( f_1 \) and \( f_2 \). Now since the test is being performed on the same physical device, \( k_{p}, k_{e}, t \), the number of turns \( n_p \), and the volume of the core and the area of the core are the same for both tests. Since (7.8) can be solved for \( B_{Max} \) to give

\[
B_{Max} = \frac{\sqrt{2} \ E_p}{n_p \ 2 \ \pi \ f \ A_c}
\]  

(AII-6)

we see that if we adjust the frequency \( f \), and the voltage \( E_p \) by the same amount, then the flux \( B_{Max} \) remains constant. If we set

\[
P_{Loss} = P_h + P_e
\]

\[
K_1 = k_h (B_{Max})^x \ Vol_{Core}
\]

\[
K_2 = k_e t^2 (B_{Max})^2 \ Vol_{Core}
\]

(AII-7)

Then (7.47) becomes

\[
P_{Loss} = K_1 f + K_2 f^2
\]

(AII-8)

Note that while \( K_1 \) and \( K_2 \) are the same for the tests at both frequencies (if we decrease the excitation voltage \( E_p \) by the same amount that we decrease the frequency \( f \)), we cannot determine their values since we don’t know \( k_{p}, k_{e}, n_p, A_c, \) or \( Vol_{Core} \). Now it would be useful to know \( K_1 \) and \( K_2 \), since

\[
P_h = K_1 f
\]

\[
P_e = K_2 f^2
\]

(AII-9)

The procedure for solving (AII-8) for \( K_1 \) and \( K_2 \) is to run the open circuit test twice, once at frequency \( f_1 \) and voltage \( E_1 \) yielding \( P_1 \), and again at \( f_2 \) and \( E_2 \) (where \( f_2 = k f_1 \) and \( E_2 = k E_1 \)) yielding \( P_2 \). Then the system

\[
P_1 = K_1 f_1 + K_2 f_1^2
\]

\[
P_2 = K_1 f_2 + K_2 f_2^2
\]

(AII-10)

is easily solved for \( K_1 \) and \( K_2 \), since \( f_1 \) and \( f_2 \) are known, and \( P_1 \), and \( P_2 \) are measured.

Then the loss in \( \left( \frac{W}{m^3} \right) \) in (AII-4) is multiplied not by \( \frac{1}{2} \) (which supposes that the core losses split evenly between \( P_h \) and \( P_e \)), but by \( \frac{P_h}{P_{h_1} + P_{e_1}} \), and the loss in \( \left( \frac{W}{m^3} \right) \) in (AII-5) is

7-34
multiplied by \( \frac{P_{e_1}}{P_{h_1} + P_{e_1}} \), where \( f_1 \) is the rated frequency of the transformer and \( f_2 = \frac{f_1}{2} \).

For example, suppose that we have a transformer wound on M6 grain oriented electrical steel (this is the material we used in our transformer design example), and we run the open circuit test at 60 Hz (at full voltage) giving \( P_1 = 500\text{W} \), and at 30 Hz (at half voltage) giving \( P_2 = 200\text{W} \). (Notice that this test transformer need not bear any relationship to the transformer we are trying to design - any transformer wound on the material we are testing will do, as long as the voltage rating is the same as the voltage rating of the transformer we are trying to design.) Then (AII-10) becomes

\[
\begin{align*}
500 &= K_1 (60) + K_2 (60)^2 \\
200 &= K_1 (30) + K_2 (30)^2
\end{align*}
\]

which is easily solved by Cramer’s Rule for \( K_1 = 5.00 \) and \( K_2 = \frac{1}{18} \). Then \( P_{h_1} = 300\text{W} \) and \( P_{e_1} = 200\text{W} \) and (AII-4) becomes

\[
11679.89 \left( \frac{3}{5} \right) = 114.8 \ k_h - k_h = 61.045
\]

and (AII-5) becomes

\[
11679.89 \left( \frac{2}{5} \right) = 9.923 \times 10^{-4} \ k_e - k_e = 4.708 \times 10^6
\]
APPENDIX III

NOTES ON DESIGNING TRANSFORMERS FOR SWITCH MODE POWER SUPPLIES

SWITCH MODE POWER SUPPLIES
Modern electronic power supplies are no longer typically designed as linear power supplies. As you may recall from courses in electronics, a linear power supply is one which uses a 60 hertz transformer to change the voltage level from 115 VAC to an AC voltage whose peak value is a few volts higher than the DC voltage we are trying to produce. The transformed voltage is rectified, filtered, and regulated before being connected to the DC load that the power supply is intended to supply. The linear supply has fallen out of favor for two reasons. First, the 60 hertz transformer is heavy and expensive, and second, the linear regulator is not very efficient. The modern approach is to rectify (and filter) the 115VAC directly and use a DC / DC converter to change the voltage magnitude to the required level. This conversion is done by using a transistor to turn the rectified and filtered 115V (this is a DC voltage) on and off at a relatively high (100kHz - 10 MHz) frequency (this is called chopping), and pulse width modulating the chopped waveform to produce a voltage whose average value is the value of the DC we are trying to produce. The chopped, modulated waveform is then smoothed (filtered) to produce low ripple DC. Power supplies that operate in this manner are called Switch Mode Power Supplies (SMPSs).

CORE MATERIALS
Transformers for SMPSs are usually not wound on grain oriented electrical steel but on a ferromagnetic ceramic called ferrite. This material, consisting largely of oxides of iron and manganese carefully processed into a ceramic material, exhibits very high resistivity (very low conductivity). Since the resistivity can reach $10^7 - 10^8 \, \Omega \cdot \text{cm}$, the eddy current losses of cores made of ferrites are negligible. This advantage coupled with the low density (and therefore low weight) make ferrite the material of choice for SMPS transformers. There is a disadvantage to these materials, and that is their relatively low saturation flux density (typically less than about 0.3Tesla). Nevertheless ferrites are currently the most common core material used in SMPSs.

WIRE SIZE
Since SMPSs typically operate at frequencies between several tens of kilohertz to several megahertz, the skin effect becomes important for these transformers and influences the choice of wire size and manner of winding of these devices. As you may recall from electromagnetic field theory, the AC current flowing in a round conductor is confined to an annulus at the circumference of the conductor as shown in Fig. AIII-1. The thickness of the anulus is called the skin depth $\delta$ and is a function of frequency as given in (AIII-1).
Figure AIII - 1

\[ \delta = \sqrt{\frac{2 \rho}{\omega \mu}} \]  

(AIII-1)

where \( \mu \) is the relative permeability (which, for copper is approximately 1.0), and \( \rho \) is the resistivity which is given by (7.43).

We can see that if the diameter of the wire is less than or equal to twice the skin depth, that the current flows nearly uniformly over the whole conductor area and the skin effect can be ignored.

The usual procedure is to calculate the required diameter by (7.58) or (7.59), then calculate the skin depth at the SMPS operating frequency, and use a sufficient number of conductors whose diameter is 2 \( \delta \) connected in parallel to reach the required diameter. Eddy current effects in the paralleled winding conductors (and other effects caused by the coupled magnetic fields of the conductors) can be minimized by twisting the conductors together so that the magnetic fields of the individual conductors cancel one another out. When the number of paralleled conductors is high, the wires must be woven together so that each wire finds itself alternately near the interior of the bundle then near the exterior of the bundle. Windings constructed in this way are called Litz wire (after the inventor) and cause the window utilization factor \( k_w \) to be reduced from the value it would have for untwisted wire because the twists must be accommodated.